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FREE CASH FLOW AND INVESTMENT PRECOMMITMENT

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Abstract

Theories of high leverage based on the argument that debt repayment forces management to disgorge free cash flow ignore the fact that firms' investment decisions often affect the investment decisions of competitors. This paper shows that when firms interact through their investment decisions, free cash flow in the hands of an investment-prone manager can serve as a strategic weapon. Endowing management with free cash flow precommits the firm to large, predatory investments, which keep rivals away from common investment opportunities. The focus on the precommitment value of free cash flow leads to a number of implications on cross-firm effects that are broadly consistent with existing evidence.

I. Introduction

Agency-based arguments stress the benefits of limiting the amount of resources available to a manager who pursues his own objectives. For example, Jensen (1989) argues that high leverage disciplines growth-biased managers by forcing payment of funds that would otherwise be spent on empire-building projects. Recently, a number of models have shown that limiting the amount of resources under management control in the presence of informational asymmetries can also reduce firm value. The adverse effect arises from the incentive faced by a manager who privately observes cash flow, to underreport cash flow, appropriate the surplus for personal benefit, and "cry wolf" to security holders. In Stulz (1991), this incentive leads the firm to pass up profitable projects when cash available to the manager is low, because shareholders don't believe the manager's claim that available funds are insufficient to finance all profitable projects. In Bolton and Sharfstein (1990), this same incentive leads suppliers of capital to tie refinancing to a cash payout, rendering the firm vulnerable to predation by competitors.

This paper shows that limiting discretionary funds available to management can be costly even when information is symmetric. Curtailing discretionary funds hurts the firm in two fundamental ways: first, it limits its ability to launch large, predatory investments aimed at excluding its competition from investment opportunities; second, it encourages rivals to attempt to gain exclusive control of investment opportunities.

To develop intuition for this result consider two firms, A and B, that compete for the same project, and as a result, face a simple investment externality: an increase in the investment level of one firm reduces the profitability of investing to the other firm. Suppose the owners of firm A control the investment policy of the firm. Their policy is to invest to maximize NPV given the investment made by firm B. Given this investment policy of firm A, firm B can increase its value by hiring an investment-prone manager, endowing him with discretionary funds, and disseminating equity ownership widely to make subsequent collective action by the shareholders costly. These actions precommit firm B to a large investment in the project and, given a sufficiently adverse cross-firm impact on profitability, induce the owners of firm A to quit the project. To deter this type of predatory investment, the owners of firm A need to relinquish control over the investment policy to an agent who has the resources and the motivation to fund unprofitable investments.

I illustrate these ideas in the context of a simple duopoly model with negative investment externalities of the type described in the previous paragraph. Both firms are run by growth-biased managers who invest as much as they can. Managers have complete discretion over internal funds (consisting of current earnings net of current promised payments to creditors) but they need shareholders' approval to raise external funds. In contrast with the models of Stulz and Bolton-Sharfstein, information is symmetric to everyone.

When investment externalities are ignored, firm value is maximized with high leverage since debt payments force management to payout internal funds that might otherwise be wasted in excessive investment. When investment externalities are taken into account, however, leverage is generally set more conservatively. A critical issue is whether investments are strategic substitutes or strategic complements, i.e., whether the NPV-maximizing response to a marginal increase in investment by a rival is to lower or to raise the firm's own level of investment. Investment externalities favor equity financing unambiguously when investments are strategic substitutes but have an ambiguous effect on financial policy when investments are strategic complements. When the benefits of excluding the rival from investment opportunities are large enough, however, investment externalities favor equity financing in either case.

The analysis indicates that the usual concept of free cash flow - the amount of cash left after all NPV>0 investments have been funded - is inadequate in the presence of investment externalities. When firms compete for the same investment opportunities, the amount of funds that can profitably be invested by one firm depends on the investments made by competitors. Extending the concept of free cash flow to the case of investment externalities thus requires an equilibrium concept whereby all competitors simultaneously maximize the NPV of investing in the common opportunities. A natural extension of the concept of free cash flow is the following: The cash left after the firm - and every one of its competitors - has funded the level of investment that maximizes the NPV of investing in the common opportunities [1]. This is the concept of free cash flow used in this paper. The key result of the paper can now be summarized as follows: When firms compete for the same investment opportunities, free cash flow precommits the firm to its investment opportunities.

An alternative mechanism to precommit the firm's investment policy is through early investment (Spence (1977), Dixit (1980)). Precommiting to an investment opportunity through early investment, however, raises a number of difficulties. For example, when the nature of the investment opportunity is uncertain, it's hard for the firm to identify the right investment to make ahead of time. Also, many investments experience rapid economic depreciation making it costly to precommit through early investment (e.g. investment in advertising or in a training program). Finally, an early investment can't precommit the firm if the investment is reversible. Precommiting with free cash flow, in contrast, resolves these difficulties.

The paper contributes to the growing literature that studies the effects of financing decisions on the strategic interaction among competitors. The analysis presents similarities with the work of Bolton and Sharfstein (1990) and can be affiliated with the "deep-pocket" theory of predation. A key distinction between my model and that of Bolton and Scharfstein is the role played by the agency conflict between management and owners. In their model, ameliorating the agency conflict between management and owners tends to reduce predation, since it makes it easier for outsiders to believe the cash flow figures reported by the manager. The opposite implication is derived in my model. Ameliorating the agency conflict undermines management's willingness to invest in unprofitable projects and consequently, elicits predatory investments by rivals [2].

The rest of the paper is organized as follows: Section II presents the basic model. Section III derives cross-firm implications. Section IV illustrates the principles developed in the paper by reviewing the Massey-Ferguson Ltd. (1980) case study. Section V concludes.

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[1] Thus overinvestment is what the firm invests over and above what it would invest were all firms maximizing NPV.

[2] The two models therefore yield opposite implications about the effect of ameliorating the conflict of goals between management and owners on the value of competitors. This suggests the following test to distinguish the two models: Collect a sample of firms which announce a shift in their compensation policy toward incentive-based compensation and look at the stock price reaction of competitors.

II. The model

II.1. Assumptions and notation

The basic model is adapted from Stulz (1990). Consider a two-period duopoly with dates and firms indexed, respectively, t=0,1,2 and i=A,B. Firms start with assets in place which produce a liquidating cash-flow, Z_i (i=A,B), at t=1; the cash-flow from assets in place is a random variable distributed according to a symmetric bivariate density $g(Z_A,Z_B)$, where $0 \leq Z_i \leq +\infty$ (i=A,B). At t=1, firms invest in a common investment opportunity. The investment pays off at t=2, at which date firms liquidate and distribute all funds to security holders. The cash distribution at t=2 is $Y_i(I_A,I_B)$, where $I_i \geq 0$ is the amount invested by firm i (i=A,B) at t=1; $Y_i(.)$ is assumed to exhibit the following properties (i=A,B; j=A,B; i≠j):

$$Y_i(I_i=k,I_j=l)=Y_j(I_j=k,I_i=l), \tag{1}$$
$$Y_i(I_i=0,I_j)=0, \tag{2}$$
$$dY_i/dI_i>0, \tag{3}$$
$$dY_i/dI_j<0. \tag{4}$$

(1) assumes that the investment technology is symmetric for the two firms; (2) states that firms that don't invest, distribute no cash at t=2; (3) establishes that firms that invest more make larger distributions; finally, (4) creates a negative investment externality between the two firms.

The return to investment is specified to change from increasing returns (i.e., $d^2Y_i/dI_i^2>0$) to decreasing returns (i.e., $d^2Y_i/dI_i^2<0$) as investment is increased. The assumption that investment is made, initially, at increasing returns implies that there is minimum efficient level of investment; the assumption of decreasing returns at higher investment levels, on the other hand, guarantees that overinvestment is possible.

Firms are run by a professional manager whose utility is a function of date 1 investment only [3]. This is the same assumption found in Stulz (1990) and Hart and Moore (1992). It captures in a simple fashion the essential management bias toward corporate growth. Managers enjoy complete discretion over internal funds, subject only to the constraint of satisfying their firms' current obligations to debtholders; failure to satisfy creditors entails loss of control and a substantial utility loss. These assumptions imply that managers will invest at least $\text{Max}\{0,Z_i-F_i\}$, where F_i represents the payment due to creditors. Furthermore, managers will fund additional investment by raising as much external capital as they possibly can. I assume, as in Stulz, that managers raise funds at t=1 only if shareholders approve.

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[3] As in Stulz, management is paid a fixed wage and has no personal wealth tied in the firm. Both Y_i and Z_i are defined net of payments to the manager.

This framework implicitly assumes that stock ownership is sufficiently diffuse to make a take-over that curtails internally funded investment, prohibitively expensive. By the same token, atomistic shareholders cannot form a coalition to force management to pay out internal funds. I´m also assuming that contractual solutions linking management's pay to the amount of internal funds or to the level of investment are ineffectual [4].

The decision facing the owners at t=0 is to choose the promised payment to debtholders at t=1, F_i, that maximizes firm value. Throughout the paper it is assumed that bankruptcy is costless to the firm. If default occurs at t=1, bondholders simply take over and fund the investment level that maximizes NPV given the investment of the rival. Finally, assume that capital structure choices are made through pure exchange offers, all participants are risk neutral, and there is no discounting.

II.2. Optimal capital structure

First consider the investment policy of the firm under the principal's control. The principal I am referring to are the shareholders in the non-default states and the creditors in the default states. With the principal in control, the firm funds the level of investment that maximizes NPV given the investment of the rival [5]. The principal's problem is to solve

$$\max_{I_i} \; Y_i(I_i,I_j)-I_i \qquad \text{s.t. } I_i\geq 0 \tag{5}$$

given $I_j\geq 0$. The solution to the principal's problem generates the reaction function $I_i=R_i^P(I_j)$, relating the firm's NPV-maximizing investment level to the level of investment made by the rival. Superscript P indicates that the reaction function pertains to the principal. The specification of the returns to investment implies a discontinuity in the reaction function at the point where the investment of the rival becomes so large that is no longer possible for the firm to invest at a profit. When the rival invests above this threshold, denoted by I_j^{i0}, the firm maximizes NPV by not investing at all (thus $R_i^P(I_j)=0$ when $I_j>I_j^{i0}$). Intersections of the reaction functions of the principals of the two firms generate the Nash equilibrium NPV-maximizing investment pairs. For simplicity, I assume that the two reaction

[4] For example, if investment is not verifiable, courts cannot enforce investment-contingent contracts.

[5] Given the assumptions, there is no underinvestment problem in the sense of Myers (1977). When Z_i-F_i<0, shareholders authorize management to raise capital to fund the NPV-maximizing level of investment and payout creditors, provided NPV exceeds -(Z_i-F_i); if NPV is less than -(Z_i-F_i), the firm defaults, and bondholders take over and fund the NPV- maximizing investment level.

functions intersect only once and denote the equilibrium investment pair (I_A^P, I_B^P; $0<I_A^P<I_A^{B0}, 0<I_B^P<I_B^{A0}$). Given the symmetry in investment technology, it follows that $I_A^P=I_B^P$ and $I_A^{B0}=I_B^{A0}$. In the rest of the paper, I refer to free cash flow as the amount of internal funds in excess of I_i^P (i=A,B); I also refer to overinvestment as an investment above I_i^P (i=A,B). Thus overinvestment in this model is what the firm invests above what it would invest if the two firms were simultaneously maximizing NPV.

The amount of internal funds available to the manager for discretionary investment is Max{0,Z_i-F_i}. Since the manager invests all available internal funds plus as much external funds as shareholders authorize him to raise, the amount invested by firm i at t=1 is

$$I_i=R_i(I_j,Z_i-F_i)=\mathrm{Max}\{\mathrm{Max}\{0,Z_i-F_i\},R_i^P(I_j)\}=\mathrm{Max}\{Z_i-F_i,R_i^P(I_j)\} \tag{6}$$

where the last equality follows from the fact that $R_i^P(I_j)\geq 0$. I call the function $R_i(I_j,Z_i-F_i)$ the firm's reaction function. Notice that the firm's reaction function is the same as that of the principal when the firm's NPV-maximizing investment level exceeds available internal funds (i.e., when $R_i^P(I_j)>Z_i-F_i$). This is because the principal authorizes the manager to raise additional funds when he finds that internal funds are insufficient to finance the NPV-maximizing investment. Thus, when $Z_i-F_i<R_i^P(I_j)$, the principal is effectively in control of the investment policy since the firm funds the NPV-maximizing outlay; in contrast, when $Z_i-F_i\geq R_i^P(I_j)$, the manager is in control and the firm overinvests.

Figures 1 and 2 plot $R_i^P(I_j)$ and $R_i(I_j,Z_i-F_i)$ when $\partial^2Y_i/\partial I_i\partial I_j$ is, respectively, negative and positive. Following the terminology of Bullow, Geanakopolos, and Klemperer (1985), investment decisions are said to be strategic substitutes in the first case and strategic complements in the second [6,7]. In both plots, the bold kinked lines represent the firms' reaction functions, $R_i(I_j,Z_i-F_i)$ (i=A,B;j=A,B;i≠j); the two functions intersect at $I_A^*(Z_A-F_A,Z_B-F_B),I_B^*(Z_A-F_A,Z_B-F_B)$, which is therefore the Nash equilibrium investment pair. The plots also depict the locus of investment pairs that make NPV equal to zero for each firm.

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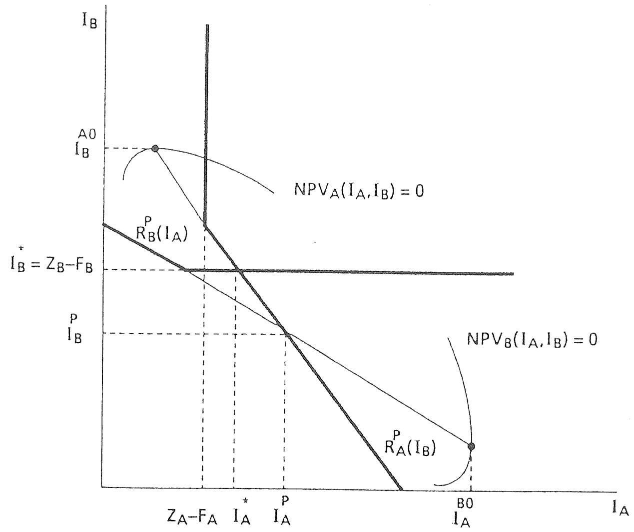
[6] Investments are said to be strategic substitutes or strategic complements when the NPV-maximizing response to a marginal increase in investment by the rival is, respectively, to lower or to raise the firm's own level of investment.

[7] Sundaran and John (1992) argue that the sign of the second-order cross-derivative in shareholders' maximand often plays a key role in models of product market competition. For example, they show that changing the sign of the second-order cross-derivative in the models of Maksimovic (1990) and Brander and Lewis (1986) either reverses or produces indeterminate results. As we shall briefly see, their remark certainly applies to the model developed here.

FIGURE 1

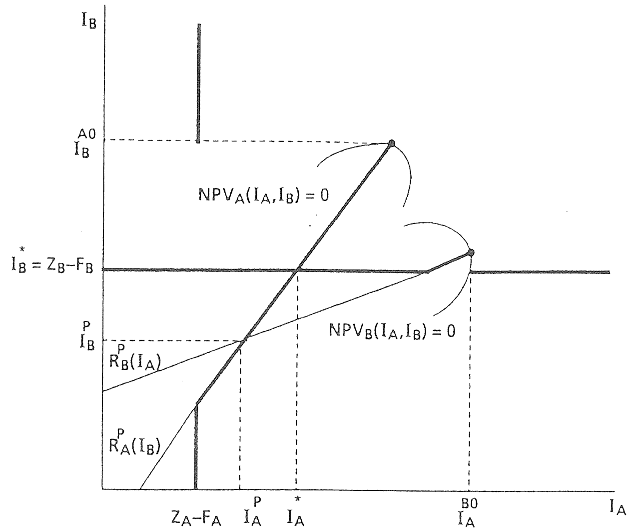
Equilibrium investment decisions when investments are strategic substitutes



$Z_i - F_i$ (i = A,B) is the amount of internal funds available to firm i; R^P_i (I_j) (i = A,B; j = A,B; i ≠ j) is the reaction function of the principal of firm i; NPV_i $(I_A,I_B) = 0$ is the locus of investment pairs that makes the net present value of the investment opportunity equal to zero for firm i; (I_A^*, I_B^*) is the Nash equilibrium investment pair.

FIGURE 2

Equilibrium investment decisions when investments are strategic complements



$Z_i - F_i$ (i = A,B) is the amount of internal funds available to firm i; R^P_i (I_j) (i = A,B; j = A,B; i ≠ j) is the reaction function of the principal of firm i; NPV_i $(I_A,I_B) = 0$ is the locus of investment pairs that makes the net present value of the investment opportunity equal to zero for firm i; (I_A^*, I_B^*) is the Nash equilibrium investment pair.

When the owners set F_i at t=0, they look at the rival's current choice of F_j to infer the rival's future reaction function. They then attempt to pre-position their own reaction function through their choice of F_i to maximize firm value. The value of the firm, at t=0, is

$$
\begin{aligned}
V_i &= E[Z_i]+E\{Y_i[I_i^*(Z_i-F_i,Z_j-F_j),I_j^*(Z_i-F_i,Z_j-F_j)]\}-E[I_i^*(Z_i-F_i,Z_j-F_j)] \\
&= E[Z_i]+\{Y_i[I_i^P,I_j^P]-I_i^P\}G(I_i^P+F_i,I_j^P+F_j) \\
&+ \int_{I_i^P+F_i}^{+\infty}\int_{0}^{R_j^P(Z_i-F_i)+F_j} \{Y_i[Z_i-F_i,R_j^P(Z_i-F_i)]-(Z_i-F_i)\}g(.)dZ_jdZ_i \\
&+ \int_{I_i^P+F_i}^{+\infty}\int_{R_j^P(Z_i-F_i)+F_j}^{Z_i-F_i+F_j} \{Y_i[Z_i-F_i,Z_j-F_j]-(Z_i-F_i)\}g(.)dZ_jdZ_i \\
&+ \int_{I_j^P+F_j}^{+\infty}\int_{0}^{R_i^P(Z_j-F_j)+F_i} \{Y_i[R_i^P(Z_j-F_j),Z_j-F_j]-R_i^P(Z_j-F_j)\}g(.)dZ_idZ_j \\
&+ \int_{I_j^P+F_j}^{+\infty}\int_{R_i^P(Z_j-F_j)+F_i}^{Z_j-F_j+F_i} \{Y_i[Z_i-F_i,Z_j-F_j]-(Z_i-F_i)\}g(.)dZ_idZ_j \qquad (7)
\end{aligned}
$$

where G(.) is the cumulative bivariate of (Z_i,Z_j). The first term in (7) is simply the expected liquidation value of assets in place. The second is the firm value when the last dollar invested by either firm is financed externally and therefore authorized by the respective principals. The third term accounts for firm value when the last dollar invested by firm i is financed internally while the last dollar invested by j is financed externally. The reverse is captured in the fourth term. Finally, the fifth term reflects firm value when the last dollar invested by either firm is funded internally.

Based on (7), the first-order condition (FOC) is

$$
\begin{aligned}
dV_i/dF_i &= \int_{I_i^P+F_i}^{+\infty}\int_{0}^{R_j^P(Z_i-F_i)+F_j} \{1-\partial Y_i[Z_i-F_i,R_j^P(Z_i-F_i)]/\partial I_i\}g(.)dZ_jdZ_i \\
&+ \int_{I_i^P+F_i}^{+\infty}\int_{R_j^P(Z_i-F_i)+F_j}^{Z_i-F_i+F_j} \{1-\partial Y_i[Z_i-F_i,Z_j-F_j]/\partial I_i\}g(.)dZ_jdZ_i
\end{aligned}
$$

$$+ \int_{I_j^P+F_j}^{+\infty} \int_{R_i^P(Z_j-F_j)+F_i}^{Z_j-F_j+F_i} \{1-\partial Y_i[Z_i-F_i,Z_j-F_j]/\partial I_i\} g(.) dZ_i dZ_j$$

$$- \int_{I_i^P+F_i}^{I_i^{j0}+F_i} \int_{0}^{R_j^P(Z_i-F_i)+F_j} \{\partial Y_i[Z_i-F_i,R_j^P(Z_i-F_i)]/\partial I_j\}(dR_j^P/dI_i) g(.) dZ_j dZ_i$$

$$- \int_{0}^{R_j^P(I_i^{j0})+F_j} \{Y_i[I_i^{j0},Max(0,Z_j-F_j)]-Y_i[I_i^{j0},R_j^P(I_i^{j0})]\} g(I_i^{j0}+F_i,.) dZ_j=0 \qquad \textbf{(8)}$$

The first three terms of the FOC consist of the direct effect whereas the last two terms reflect the strategic effect resulting from the investment externality. Since the first three terms are positive, the optimal capital structure at t=0 consists exclusively of debt (i.e., $F_i^*=+\infty$) if there is no strategic effect. This case illustrates the standard free cash flow argument for high leverage. With no bankruptcy costs and with creditors taking over and investing to maximize NPV in the default states, the solution to the overinvestment problem is a financial policy that guarantees default at t=1. When there is a strategic effect, however, the optimal financial policy can be an interior solution if the strategic effect is negative. Let $F_i^*(F_j)$ denote the optimal amount of debt when the rival's financing structure is F_j. The intersections of functions $F_i^*(F_j)$ and $F_j^*(F_i)$ determine the Nash equilibrium pairs of capital structures at t=0.[8]

II.2. *A. Strategic effects*

The strategic effect results from the influence of the firm's own investment decision on the investment choice of the rival. The fourth term of the FOC measures the impact of F_i on firm value when entry is accommodated. The *entry accommodation* effect occurs when the rival readjusts his level of investment locally in response to a marginal increase in the level of investment made by the firm. The last term of the FOC shows the effect of F_i on firm value due to entry deterrence, i.e., resulting from the influence of the firm's own investment level on the rival's decision to pass up the investment opportunity. This effect is usually referred to as the *entry deterrence* effect.[9,10]

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[8] There are as many intersections as there are fixed points for the function $F_i^*[F_j^*(F_i)]$. For example, with $F_i^*(F_j)$ non-increasing in F_j, a set of sufficient conditions for existence is that $F_i^*(F_j=+\infty)<+\infty$ and $F_i^*(F_j=0)=+\infty$. Section II.3. examines a simplified model that satisfies these conditions.

[9] If the firms were already investing in the common opportunity at t=0, then instead of an *entry deterrence* effect we would have an *exit inducement* effect. The two effects are identical for the purposes of the analysis.

[10] The terminology is borrowed from Tirole (1988).

When investments are strategic substitutes, the reaction function of the principal slopes downward making the *entry accommodation* negative. The *entry deterrence* effect is also negative since the last term of the FOC is always negative. Thus overall, the strategic effect is negative and tends to offset the direct effect. A sufficient condition for equity financing to exist in equilibrium is for the strategic effect to dominate the direct effect when the rival is highly levered (i.e., that $F_i^*(F_j=+\infty)$ is finite).

When investments are strategic complements, the reaction function of the principal slopes upward. This implies a positive *entry accommodation* effect, which reinforces the direct effect. The *entry deterrence* effect continues to be negative. Equilibria capital structures will contain equity if the *entry deterrence* effect dominates the direct plus the *entry accommodation* effect when the rival adopts a high level of debt.

In summary, the strategic effect has the potential to lead to equilibrium where free cash flow is optimally allocated to the manager.[11] Such equilibrium is more likely when investments are strategic substitutes since, in that case, the strategic value of free cash flow is unambiguously positive. In the case investments are strategic complements, equilibrium will also feature free cash flow if the benefits from entry deterrence (or from exit inducement) are sufficiently large.[12]

The *entry deterrence* effect is stronger when the rival is highly levered, as shown by the negative impact of F_j on the last term of the FOC. A highly levered rival has his investment policy controlled by the principal in more states of nature, and therefore is easier to be induced to stay out of an investment opportunity by confronting him with a predatory investment. This property of the *entry deterrence* effect implies that high leverage encourages competitors to attempt to gain exclusive control of investment opportunities through large, internally financed investments. A conservative debt policy, on the other hand, discourages this type of predation since it commits the firm to invest.

II.3. *A simplified model*

To study more closely equilibrium financing choices, this section examines a simplified model where the strategic effect consists solely of an *entry deterrence* effect. This special case is simple enough to allow for financing equilibria to be explicitly derived in the presence of strategic interaction between rivals.

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[11] The model reflects, essentially, a simple prisoners' dilemma situation. Given that the rival is highly leveraged, it is optimal for the firm to keep leverage low and provide management with discretionary resources to fund corporate growth. This suggests an extension of the model where firms use leverage, in a dynamic collusive equilibrium, to commit to a non-expansionary investment policy. An example of a collusive equilibrium along these lines is given by Maksimovic (1988).

[12] In either case, many plausible specifications of the expected investment payoff, $Y_i(.)$, and the bivariate density g(.,.) can be found that generate interior capital structures

Suppose that the investment payoff function in the range of decreasing returns is of the form

$$y_i(I_i,I_j) = m\,(I_i) + h\,(I_j) \qquad \textbf{(9)}$$

Under (9) the NPV-maximizing level of investment is a constant magnitude, I^*, independent of I_j, therefore limiting the influence of the investment of the rival on the firm's own investment decision to an *entry deterrence* effect.[13]

The *entry deterrence* effect operates when the investment of the rival reaches a threshold where the firm's own NPV-maximizing action is not to invest. This threshold is given by the value of I_j that solves $y_i(I_i=I^*,I_j)-I^*=0$. I refer to this threshold as the entry deterrence investment and denote it by I^0. The NPV-maximizing investment is zero when the rival invests I^0 (or more) and I^* otherwise. Given that $I^0>I^*$, it follows that equilibrium outlays when firms' investment policies are under the principals' control, are unique and characterized by both firms investing I^*.

To derive equilibrium financing decisions at date t=0, it's necessary to parameterize the bivariate distribution of cash-flow, $g(Z_A,Z_B)$. I assume that there are four possible outcomes for the pair (Z_A,Z_B): (Ω,Ω) with probability p_{uu}, $(\Omega,0)$ with probability p_{ud}, $(0,\Omega)$ with probability p_{du}, and (0,0) with probability p_{dd}. To maintain symmetry in the bivariate distribution it is assumed that $p_{ud}=p_{du}$. Also, it is assumed that $\Omega \geq I^0$ so that cash flow in the upstate exceeds the investment outlay necessary to deter entry. Finally, assume that there are arbitrarily small flotation costs. Everything else the same, the firm chooses the capital structure that minimizes future external financing. This assumption guarantees that the optimal capital structure is unique for every possible capital structure of the rival.

In this simplified model there are only three reasonable financing choices, $F_i=\Omega$, $F_i=\Omega-I^*$, and $F_i=\Omega-I^0$, which I call, respectively, high, moderate, and low leverage (or equivalently, high, moderate, and low debt ratios). The high leverage alternative forces management to payout all available internal funds and thus guarantees that the firm will follow the NPV-maximizing investment policy. The moderate leverage alternative endows management in the cash flow upstate with I^* of discretionary funds, an amount that is equal to the NPV-maximizing outlay when the rival invests below the entry deterrence threshold. Hence, if the leverage of the rival is moderate or high, financing choice $F_i=\Omega-I^*$ is equivalent to choice $F_i=\Omega$ except for lower flotation costs; on the other hand, when the leverage of the rival is low, $F_i=\Omega$ dominates $F_i=\Omega-I^*$ since it keeps the firm out of the project when the investment of the rival makes entry unprofitable. The low leverage alternative, $F_i=\Omega-I^0$, provides the manager in the upstate with just enough

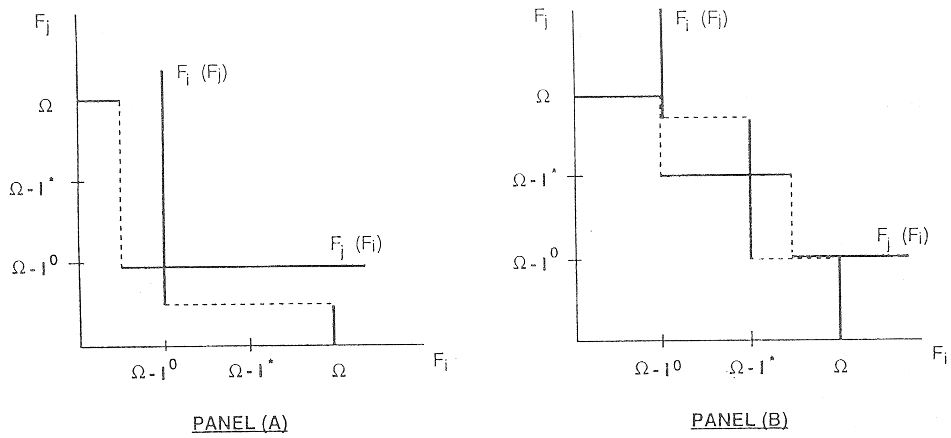
[13] For example, if the common investment opportunity available to the two firms requires non-divisible investments of fixed size in units of productive capacity and an investment produces negative marginal returns beyond one unit of capacity, there is no *entry accommodation* effect.

discretionary funds to attempt to deter the rival from entering. All other alternative financing choices lead either to unnecessary overinvestment or/and excessive flotation costs.

Figure 3 depicts optimal financing policies and the resulting Nash equilibrium pairs of capital structures for different values of the exogenous parameters. The details of the derivation are contained in appendix A. Here, I simply provide an intuitive discussion.

FIGURE 3

Financing equilibria of simplified model



Panel (A): $p_{ud}\,[Y_i\,(I_i^*, I^*) - Y_i\,(I_i^0, 0) - I^* + I^0] - p_{uu}\,[\gamma\,(I^0) - \gamma\,(I^*) + I^* - I^0] < 0$

Panel (B): $p_{ud}\,[Y_i\,(I_i^*, I^*) - Y_i\,(I_i^0, 0) - I^* + I^0] - p_{uu}\,[\gamma\,(I^0) - \gamma\,(I^*) + I^* - I^0] > 0$

F_i is the amount of debt issued by Firm i; $F_i\,(F_j)$ is the optimal amount of debt issued by Firm i given that Firm j issues F_j; I^0 is the entry deterrence investment threshold; I^* is the Nash equilibrium investment; Ω is the cash flow from assets in place in the upstate.

In the simplified model, a policy of low leverage maximizes the firm's own value when the rival maintains a high debt ratio. To see why, recall that a rival with a high debt ratio always invests to maximize NPV. A predatory investment launched against a highly levered rival therefore, is guaranteed to succeed in deterring entry and will certainly deliver the profits associated with exclusive control of the investment opportunity. An important corollary is that a symmetric equilibrium in high leverage is not feasible. Thus, the prescription of high debt payments as a means to eliminate discretionary resources going into the hands of growth biased managers cannot increase firm value, at the same time, for every firm in the industry.

As the rival switches to equity financing and raises its commitment to the investment opportunity, however, the firm's impetus for predatory investment diminishes, making low leverage a less desirable alternative. The nature and number of financing equilibra depend on whether low leverage remains the firm's value-maximizing choice when the rival adopts a low debt ratio.

Panel (A) of Figure 3 shows financing equilibria when low leverage is the firm's best response to a low debt ratio by the rival. The only equilibrium is a symmetric equilibrium in low leverage whereby the two firms precommit to attempt to deter entry in their cash flow upstates. From the perspective of any of the firms, the attempt to deter entry is successful when cash flow for the rival is down but leads to overinvestment when the cash flow is up since, in that case, the two firms launch mutually destructive predatory investments. The unique symmetric equilibrium in low leverage is therefore more likely when the correlation between the cash flows of the two firms is low and the benefit of exclusive access is large vis-à-vis the cost of overinvestment.

The case where low leverage is a dominated strategy against a highly levered rival is presented in panel (B) of Figure 3. There is a symmetric equilibrium in moderate leverage and two asymmetric equilibria in low-high leverage. In the symmetric equilibrium, the amount of discretionary internal funds generated by each firm in their cash flow upstate is equal to the Nash equilibrium NPV-maximizing outlay, setting a minimum investment that is large enough to deter the rival from preying. The consequence is that the investment outlays of the two firms are always consistent with the Nash equilibrium NPV-maximizing investment pair. This equilibrium provides a curious illustration of the principle that firms should try to match internal funds with profitable outlays. In the asymmetric equilibria, the firm with low leverage precommits to the entry deterrence investment while the firm with high leverage yields to the aggressive investment policy of the rival by committing to stay out when the rival attempts to deter entry. The low leverage choice in the asymmetric equilibrium achieves greater firm value than any other equilibrium financing policy and is therefore, the most desirable outcome for either firm.

II.4. *Precommitment and credibility*

It is useful to reflect upon why owners cannot precommit to the investment opportunity simply by making the level of external funding at t=1 contingent on the actions of the rival. For example, the owners could indicate their commitment to the investment opportunity by declaring their intent to advance funds in response to a predatory investment by the rival. Unfortunately, such a commitment has a fundamental weakness: it's not credible. Unless forced otherwise, the owners at t=1 will ignore their earlier intent and provide the amount of external funds that finances the NPV-maximizing level of investment. Free cash flow, on the other hand, constitutes a credible precommitment because it's discretionarily invested by a growth-biased manager.

Another potentially credible way to precommit the investment policy is to invest ahead of time (i.e., invest at date t=0). To be credible, an early investment needs to convert some of the marginal cost of investing at t=1, into sunk costs. This is a well-known result first discussed by Spence (1977) and Dixit (1980). Precommitting with an early investment, however, raises a number of difficulties.

One problem is that the exact nature of future investment opportunities is often uncertain. Hence, firms that attempt to lock in a potential market by investing in advance will frequently make the wrong investment. For example, the Unix system developed by Bell Labs or the graphical interface of Steve Jobs which many thought would set new industry standards didn't live up to the high expectations because the markets for these technologies never develop. In another case reported on the *Asian Wall Street Journal*, Mitsui and Company invested over $20 million in Gain Electronics Corporation, a US high-technology venture that sought to develop gallium arsenide computer chips. The investment never paid off, and Mitsui reported that 'it became clear that a market for gallium arsenide was unlikely to develop soon... it was a really promising technology when we went in, but the timing just wasn't right'. The Japanese company subsequently withdrew its investment from the venture.[14] Another difficulty of precommitting with an early investment is that many investments experience rapid economic depreciation. For example, an investment in advertising loses much of its impact if made well before the product goes for sale. Similarly, investing in a training program makes little sense if the firm has no immediate use for the skills developed through the program. Finally, an early investment won't precommit the firm's investment policy if the investment is reversible.

III. Cross-firm effects

This section explores cross-firm implications of the general model developed in II.1 and II.2.The focus on cross-firm effects is motivated by the desire to produce implications that are sharply distinct from those produced by models that ignore interactions among firms. I also review a number of documented empirical regularities and discuss their consistency with the predicted cross-firm effects.

One issue of interest is the effect of a change in the firm's own leverage on the average investment of the rival [i.e., $dE(I_j \mid F_i,F_j)/dF_i$]. This effect is given by the expression

$$dE(I_j \mid F_i,F_j)/dF_i = - \int_{I_i^P+F_i}^{I_i^{j0}+F_i} \int_{0}^{R_j^P(Z_i-F_i)+F_j} (dR_j^P/dI_i)g(.)dZ_j dZ_i$$

$$- \int_{0}^{R_j^P(I_i^{j0})+F_j} [Max(0,Z_j-F_j)-R_j^P(I_i^{j0})]g(I_i^{j0}+F_i,.)dZ_j \qquad (10)$$

The first term in (10) reflects the *entry accommodation* effect and its sign depends on whether investments are strategic substitutes or strategic

[14] This case is reported by Hurry, Miller, and Bowman (1992).

complements. The second term reflects the *entry deterrence* effect and is always positive. Hence, if investments are strategic substitutes an increase in the firm's own leverage induces the rival to expand. The effect is ambiguous when investments are strategic complements. If, however, the capital structure of the rival contains equity (i.e., if F_j^* is finite), the overall strategic effect is positive, implying that the rival will expand whenever the firm increases its debt ratio.[15]

A related issue is the effect of leverage on the probability of entry (or of exit) of the rival. The probability of entering the investment opportunity is simply the probability of making a positive investment at t=1 which is given by

$$\Pr\{I_j>0/F_i,F_j\}=\Pr\{Z_j-F_j>0\}+\Pr\{Z_j-F_j<0\}\Pr\{Z_i-F_i<I_i^{j0}\} \qquad \textbf{(11)}$$

Expression (11) indicates that when firm i increases its leverage, firm j is more likely to invest [i.e., $d\Pr\{I_j>0 \mid F_i,F_j\}/dF_i>0$]. Accordingly, the model predicts that firms that increase their leverage will have their investment opportunities more frequently challenged by competitors.

The firm's financial policy also generally affects the value of the rival. From expression (7) I can compute the impact of an (unexpected) increase in the leverage of firm j on the value of firm i. This is given by

$$dV_i/dF_j = -\int_0^{R_i^P(I_j^{i0})+F_i} \{Y_i[\mathrm{Max}(0,Z_i-F_i),I_j^{i0}]-(Z_i-F_i)\}g(.,I_j^{i0}+F_j)dZ_i$$

$$-\int_{I_i^P+F_i}^{+\infty}\int_{R_j^P(Z_i-F_i)+F_j}^{Z_i-F_i+F_j} (\partial Y_i/\partial I_j)g(.)dZ_j dZ_i - \int_{I_j^P+F_j}^{+\infty}\int_0^{Z_j-F_j+F_i} (\partial Y_i/\partial I_j)g(.)dZ_i dZ_j \qquad \textbf{(12)}$$

Since all terms of (12) are positive, an (unexpected) increase in leverage is always good news to the rival.

Chevalier (1992) reports empirical results that are consistent with these cross-firm effects. Her study examines local market entry and expansion decisions of large supermarket chains following an LBO by one incumbent. She finds that large supermarket chains are more likely to expand in local markets dominated by LBO incumbents. Large supermarket chains are also more likely to enter local markets dominated by LBO incumbents. In addition, she reports that supermarket chains experience a positive stock return response at the announcement of LBOs by rival chains operating in the same local markets.

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[15] On the other hand, if we look at *relative expansion*, rivals will expand under much weaker conditions. For example, a sufficient condition for the rival's share of the total investment to go up in response to an increase in the firm's own debt ratio is that $dR_j^P/dI_i \le 1$; thus, rivals will generally grow in relative size even when investments are strategic complements.

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The last cross-firm effect investigated in this section is the effect of the firm's own default on the value of the rival. Lang and Stulz (1992) conjecture that the announcement of bankruptcy will generally produce two offsetting effects on the value of competitors. It will produce a negative *contagiation* effect resulting from the adverse impact of the bankruptcy announcement on investors' expectations of earnings for competitors. It will also produce a positive *competitive* effect since competitors improve their competitive position at the expense of a bankrupt firm. Because the two effects are of opposite signs, the authors argue that the effect of bankruptcy on the value of rivals cannot *a priori* be signed. Nevertheless, they argue that the effect of bankruptcy on competitors should be positively related to proxies for the degree of strategic interactions within the industry, a conjecture that is borne out by their data. The authors report that the average abnormal return on a portfolio of competitors' equities is positive for high-Herfindahl index industries but negative for low and moderate-Herfindahl index industries.

The current model formalizes the insight of Lang and Stulz. A positive correlation between cash flows from assets in place creates a negative *contagiation* effect of bankruptcy whereas the investment externality creates a positive *competitive* effect. Bankruptcy produces a positive *competitive* effect on competitors because it gives them the assurance that their attempts to deter the (bankrupt) firm from investing in common opportunities will succeed. Just as in Lang and Stulz, the two effects tend to offset each other, rendering the effect of default on the value of the rival ambiguous. This effect, however, can be signed in two special cases. First, when cash-flows are positively correlated but there are no investment externalities, the effect is clearly negative since there is only a *contagiation* effect. This special case would seem to apply to bankruptcy announcements of firms in industries with low to moderate concentration where investment interactions among firms are, presumably, not significant. Second, when there are investment externalities but cash flows from assets in place are uncorrelated, the effect is clearly positive since, in that case, there is solely a *competitive* effect.[16] An example of this case is bankruptcy announcements in concentrated industries where bankruptcy results from management fraud.

IV. Massey-Ferguson Ltd. (1980)[17]

One example that illustrates the precommitment value of free cash flow in the presence of investment interactions is offered by the Massey-Ferguson Ltd.

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[16] Lang and Stulz (1992) also report that the stock price response of competitors to bankruptcy announcements in concentrated industries is highest when competitors have low leverage. They discuss two possible reasons to explain this finding: first, high leverage accentuates the negative *contagiation* effect (a simple financial risk argument); second, high leverage reduces the positive *competitive* effect since it limits the ability of competitors to vigorously exploit the opportunities created by bankruptcy. In the model presented here, leverage reduces the positive *competitive* effect when the optimal capital structure of the rival is an interior solution.

[17] This material is borrowed from the Harvard Business School case "Massey-Ferguson Ltd., 1980" and the accompanying teaching note prepared by Professor Carliss Baldwin.

(1980) case study. This case study examines the financial and investment policies of the top two companies in the farm equipment manufacturing industry - a highly concentrated industry with significant investment externalities. In particular, the case discusses how the financial policies of the two companies influenced their ability to compete in the key North American market.

In the 1970s Massey-Ferguson Ltd. was the second largest company in the farm equipment manufacturing industry. The company offered a full product line, which was sold throughout the world. Between 1971 and 1976 the company enjoyed a period of rapid growth with adequate profitability. Sales grew at an annual compound rate of 22%, largely by pioneering new markets for small tractors in less-developed countries. By 1976, its market share was roughly 34%.[18] The financial strategy of the firm over this period was characterized by high leverage with a target debt-to-assets ratio of about 47%.

The top player in the industry was John Deere with a market share of 38% in 1976. While also offering a full product line throughout the world, John Deere focused on the market for large tractors in developed countries, particularly the North American market. Deere also pursued a more conservative financial policy, keeping its target debt-to-assets ratio at 30%.

In the late 70s the downturn in the economy led to sharp declines in sales of big-ticket capital goods such as tractors. Foreign markets served by Massey collapsed in 78-79 and the North American market followed in 1980. With revenues dwindling and with high interest payments to service, Massey became quickly strapped for cash. To raise cash the company sold peripherical assets, closed plants, and laid off workers. In spite of the recession, the company continued to view with optimism the investment opportunities available in the farm equipment market, particularly in the North American market. Massey had turned its attention to the North American market in 1975 by launching a new range of tractors and an improved baler line. This effort was continued in 1978, with the introduction of a new line of large, high-horsepower, tractors. By 1980, the firm estimated in its annual report a range of profitable investment opportunities requiring outlays of about $500 to $700 millions spread over the following 5 years. Funding these investment opportunities, however, required external finance. Unfortunately, the company found itself unable to raise new long-term capital. Investors, including Massey's largest shareholder (Massey largest investor was Argus Corporation, which controlled 16.5% of the shares and held 6 positions of the 18 member board including the chairman seat), were reluctant to provide capital. Liquidity constrained and without external financing available, the company was forced to scale down capital expenditures. By 1980

[18] All market share figures reported in the case are calculated as the ratio of own sales to the combined sales of the top three companies in the industry. The top 3 companies controlled about 90% of total sales.

its capital expenditure share[19] was less that one fourth of what had been in 1977 and its effort to penetrate the North American market had faltered. In 1980 alone, the number of dealerships in North America fell by 50%. The investment cuts caused market share to drop to 28% by 1980. In October of 1980, Conrad Black (the CEO of Argus and chairman of the board of Massey) threw in the towel by donating its stock to Massey's pension fund.

In contrast with Massey, John Deere entered the recession with a modest amount of debt. That gave Deere higher internal cash flow which the company fully reinvested in the North American market. The unused debt capacity also allowed management to exercise its discretion over working capital policy and finance additional expansion with short-term borrowings.[20] The company built capacity and strengthened its dealership network, leading its capital expenditure share to double between 1976 and 1980. As a result, sales grew by 75% and the company's market share rose to 49%. By 1980 the company had effectively locked up the North American market.

The case illustrates how excessive leverage weakens the firm's commitment to investment opportunities and, as a result, invites rivals to make large predatory investments. Massey, by pursuing an aggressive financial policy in the good times, made its emerging North American business vulnerable to predation in an economic downturn. Deere, in contrast, chose a conservative financial policy in the good times. When the recession came Deere could muster discretionary resources to fund expansion, and seized the opportunity to drive Massey out of the North America market.

VI. Conclusion

It has been argued that leverage can serve as a bonding mechanism that controls management's proclivity to overinvest. This paper shows that when there are strategic effects of investment, a firm maintains a credible commitment to an investment opportunity by providing management with discretionary funds. In this framework, forcing management to pay out funds through high debt repayments exposes the firm to predation by rivals with ample internal resources. These ideas are developed in the context of a simple free cash flow model where two firms compete for the same investment opportunity. The model generates a number of implications on cross-firm effects that are broadly in line with existing evidence.

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[19] Capital expenditure shares figures reported in the case are defined as the company's own capital expenditure dived by the combined capital expenditures of the top three firms in the industry.

[20] In 1980, Deere's operating income after interest payments was $320 million, capital expenditures were $420 million, and short-term borrowings increased (from the previous year) by $500 million.

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APPENDIX A

In this appendix, I show that the two panels of figure 3 contain all possible optimal financing policies and resulting Nash capital structure equilibria for the simplified model of Section II.3.A.

Let the difference between the firm value under high and low leverage (i. e., financing choices $F_i=\Omega$ and $F_i=\Omega-I^0$) be denoted by $\Delta_i(F_j)$. Since flotation costs are arbitrarily small, a low debt ratio is optimal if $\Delta_i(F_j)\leq 0$; on the other hand, if $\Delta_i(F_j)>0$, a high debt ratio is optimal only when the financial structure of the rival doesn't support the entry deterrence investment. Thus, the optimal financing policy of firm i at t=0, $F_i(F_j)$ (i=A, B; j=A, B; i≠j) is

$$F_i(F_j) = \begin{cases} \Omega & \text{if } \Delta_i(F_j)>0 \text{ and } F_j\leq\Omega-I^0 \\ \Omega-I^* & \text{if } \Delta_i(F_j)>0 \text{ and } F_j>\Omega-I^0 \\ \Omega-I^0 & \text{if } \Delta_i(F_j)\leq 0 \end{cases}$$

and there are as many Nash equilibria pairs of financing policies as there are intersections of $F_A(F_B)$ and $F_B(F_A)$.

It's tedious but straightforward to show that $\Delta_i(F_j)$ is equal to:

$$\begin{cases} p_{ud}[y_i(I^*,I^*)-I^*-y_i(I^0,0)+I^0]-p_{uu}[y_i(I^0,\Omega-F_j)-I^0] & \text{if } 0\leq F_j<\Omega-I^0 \\ p_{ud}[y_i(I^*,I^*)-I^*-y_i(I^0,0)+I^0]-p_{uu}[y(I^0)-Y(I^*)+I^*-I^0] & \text{if } \Omega-I^0\leq F_j<\Omega-I^* \\ (p_{uu}+p_{ud})[y_i(I^*,I^*)-I^*]-p_{ud}[y_i(I^0,0)-I^0]-p_{uu}[y_i(I^0,\Omega-F_j)-I^0] & \text{if } \Omega-I^*\leq F_j<\Omega \\ (p_{uu}+p_{ud})[y_i(I^*,I^*)-I^*-y_i(I^0,0)+I^0] & \text{if } F_j\geq\Omega \end{cases}$$

Because $\Delta_i(F_j)$ is non-increasing in F_j and $\Delta_i(F_j\geq\Omega)<0$, function $\Delta_i(F_j)$ changes sign at most once. When $\Delta_i(F_j=0)\leq 0$, $\Delta_i(F_j)$ is negative throughout the entire range and thus the optimal financing policy is always $F_i=\Omega-I^0$. Symmetry then implies that $(\Omega-I^0,\Omega-I^0)$ is the only Nash equilibrium pair of financing choices. When $\Delta_i(F_j=0)>0$, $\Delta_i(F_j)$ changes from positive to negative as F_j goes from zero to Ω. The number and configuration of Nash equilibria, in this case, depend on the sign of $\Delta_i(\Omega-I^0\leq F_j<\Omega-I^*)$, an expression that is independent of F_j. If $\Delta_i(\Omega-I^0\leq F_j<\Omega-I^*)$ is negative, the change in the sign of $\Delta_i(F_j)$ occurs when F_j is less than $\Omega-I^0$; the optimal financing policy is $F_i=\Omega-I^0$ when F_j is above the crossover and $F_i=\Omega$ otherwise. Hence, pair $(\Omega-I^0,\Omega-I^0)$ is, again, the only Nash equilibrium pair of financing choices. On the other hand, if $\Delta_i(\Omega-I^0\leq F_j<\Omega-I^*)$ is positive the change in the sign of $\Delta_i(F_j)$ occurs when F_j is between $\Omega-I^*$ and Ω; the optimal financing policy is $F_i=\Omega-I^0$ when F_j is above the crossover, $F_i=\Omega-I^*$ when F_j lies between the crossover and $\Omega-I^0$, and $F_i=\Omega$ when $F_j\leq\Omega-I^0$, generating three Nash equilibrium pairs: $(\Omega-I^*,\Omega-I^*)$, $(\Omega-I^0,\Omega)$ and $(\Omega,\Omega-I^0)$. In summary, there exist either one or three feasible equilibrium financing policies depending on the sign of expression $\Delta_i(\Omega-I^0\leq F_j<\Omega-I^*)=p_{ud}[y_i(I^*,I^*)-I^*-y_i(I^0,0)+I^0]-p_{uu}[m(I^0)-m(I^*)+I^*-I^0]$.