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THE COST OF EQUITY OF PORTUGUESE PUBLIC FIRMS: A DOWNSIDE RISK APPROACH

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Abstract

The most important asset pricing models consider that an investor's utility function is completely defined by mean and variance, which requires the normality of the stock's return distribution (or that stock returns are not skewed). However, not all stocks or markets have normal returns (e.g.: emerging markets and small public firms). The higher-order moments of stock return distributions (such as skewness and kurtosis) are valued by investors and need to be incorporated in the asset pricing models. In this article we analyse the normality and symmetry of a sample of Portuguese stocks and estimate their cost of equity (or the investors' required return on equity) using several measures of downside risk.

JEL: G12, G31

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1. INTRODUCTION

Under the standard discounted cash flows framework, the value of an asset (an investment project or a firm) is estimated by discounting to the present its future free cash flows using an appropriate discount rate. Traditional literature (see Brealey and Myers (2002)) defines the discount rate or the investor's opportunity cost as a weighted average of the cost of debt and the cost of equity. The cost of equity depends on the risk of the assets where the funds are applied and on the equity holder's portfolio of assets.

The equity holder's portfolio of assets defines his exposure to the different types of risk. The asset pricing model used to estimate the cost of equity (or shareholders required return) must consider these aspects and the investor's utility function. The most important (and controversial) asset pricing model is the Sharpe-Lintner Capital Asset Pricing Model (CAPM). In CAPM, an investor's utility function is completely defined by the mean and variance of portfolio returns. This is consistent with the Von Neumann-Morgenstern quadratic utility functions, and the application of this mean-variance equilibrium framework requires the normality of the stock return distribution. However, not all stocks have normal returns. For example, Peiró (1999) and Aparício and Estrada (2001) reject the normality of daily stock returns of European securities markets. The higher-order moments of a stock's return distributions (such as skewness and kurtosis) are important and valued by investors, meaning that they must be integrated in the asset pricing models.

Besides this, the CAPM assumes that all investors are diversified and, thus, systematic risk (the beta) is the only measure of risk that is valuable. Once again, this is not always true. Some investors like entrepreneurs or the stockholders of internet firms, who invest a substantial proportion of their wealth in their businesses, are not able to create a diversified portfolio and they are therefore exposed to both non-diversifiable and diversifiable risk. Within total risk there is downside risk and upside risk. Investors prefer upside risk and avoid downside risk. Stocks with positive skewness have more upside risk, and as Arditti (1975) argues, investors are willing to pay a premium for this skewness.

The main purpose of this article is to estimate the cost of equity of a sample of public Portuguese firms, using the downside risk models. This goal is achieved by the discussion and application of some asset pricing models that incorporate the effect of skewness on valuation. Since these models imply asymmetric returns, we begin by analysing the normality of stocks' return distributions, concluding that they are not normal. However, non-normality does not necessarily imply asymmetry of returns. So, we test the asymmetry of stocks' returns, concluding that skewness is significant. Given this, we apply the asset pricing models in the estimation of the cost of equity. Total risk is significantly related to stock mean returns, even more than systematic risk measures, which means that diversifiable risk is priced. On the other hand, the explanatory power of systematic skewness measures and systematic risk measures are very similar, which means that market index return distribution is symmetric. We also conclude that equity holders of sample companies are risk-loving and not risk averse.

Finally, we underline that the choice of the right risk measure to estimate the cost of equity is a function of an investor's risk exposure and of the returns distribution features of owned asset(s).

Section 2 provides an overview of downside risk models. Section 3 describes the data and methodology applied in this study. Section 4 reports the results and the empirical evidence, and finally section 5 presents the main conclusions.

2. OVERVIEW OF DOWNSIDE-RISK MODELS

Traditional literature considers that investors have a mean-variance behavior (which is consistent with the expected utility maximization) meaning either one of two conditions: (i) that an investor's utility function is quadratic; or (ii) that returns are normally distributed. Markowitz (1952) developed his theory assuming that for some utility functions, the mean-variance approximation is so good that there is virtually no room for improvement, and this is confirmed by Tsiang (1972), Levy and Markowitz (1979), Markowitz (1991) and Estrada (2004). Nevertheless, Kraus and Litzenberger (1976) conclude that investors have an aversion to variance and have a preference (and price) for positive (systematic) skewness. The authors question the validity of quadratic utility functions and reinforce the evidence that most investors have concave utility functions, displaying decreasing absolute risk aversion. An investor with a quadratic utility function displays increasing absolute risk aversion, meaning that he tends to invest less in risky assets as his wealth increases, which does not seem to be a very plausible assumption. Brennan (1979) also discusses this aspect and shows that if the market index rate of return has constant mean and volatility, the average investor has a power utility function.

Furthermore, the normality of returns is questioned by many empirical studies (e.g.: Peiró[1] (1999), Machado-Santos and Fernandes (2005), Aparício and Estrada (2001)), mainly with data from emerging markets. Estrada (2004) argues that the standard deviation is an appropriate measure of risk only when the underlying distribution of returns is symmetric and normal, otherwise it must be adapted. The author proposes semi-standard deviation as a measure of risk when the asset's distribution of returns is skewed and shows that mean-semi standard deviation behavior is an approximately-correct criterion to maximize the expected utility function.

In our opinion, when the market indexes or stocks have skewed return distributions, we cannot use mean-standard deviation models. Moreover, Markowitz (1952) had already verified this reality and argued that semi-variance seems more plausible than variance as a measure of risk. A growing

[1] Peiró (1999) refers to several authors that propose different statistical distributions for price changes of financial assets. All of these distributions reflect high kurtosis in the empirical distribution of returns (more peaked and with fatter tails than the normal distribution).

number of academics and practitioners are claiming that standard deviation and beta are not relevant measures of risk for many investment situations because they do not capture "what is at stake", principally, for low capitalization markets or stocks (e.g.: Sortino and Van der Meer (1991), Harlow (1991), Marmer and Louis (1993), Harvey (1995), Leland (1999), Nawrocki (1999)), and that is why in the last decade several authors developed models that capture the skewness in financial returns. Godfrey and Espinosa (1996) recommend two adjustments to CAPM, when used to estimate cost of equity for investments in emerging markets: add a spread to the risk free rate; and use an adjusted beta (it is defined as 60% of the ratio between an emerging market's standard deviation and the standard deviation of returns in U.S. market). Of course, these adjustments do not seem very linear and possible in all markets.

Erb, Harvey and Viskanta (1995, 1996) propose a model to estimate the cost of equity in emerging markets based on a country's credit ratings because they verify that these measures are significantly related to stock returns. In our opinion, this procedure does not seem relevant to corporate finance, since credit risk is not the major source of risk for stockholders. Damodaran (2001) develops a framework to estimate the beta of a company based on its fundamental variables. This framework could be applied to private firms as well as to emerging markets. Although academics consider the issue of skewness more relevant in emerging markets, Harvey and Siddique (2000) show that systematic skewness explains some of the cross-sectional variation of expected returns and is economically important in the U.S. stock market. A further analysis of downside risk models within the context of funds performance evaluation is in Machado-Santos (1998).

We present some of the models that capture the skewness in financial returns, and we employ them in the estimation of the cost of equity of Portuguese firms. The first two models capture the downside risk of the stocks (they should be used by an investor who holds an undiversified portfolio) while the last two models capture the systematic component of skewness (they should be used by an investor who holds a diversified portfolio when assets returns are skewed). We use these models because, in our opinion, from those available (e.g.: Hogan and Warren (1974), Harlow and Rao (1989)), they are the most complete inside the scope of this work. We do not use VAR models because their main concern is near catastrophe events.

2.1. Lower Partial Moment

Lower Partial Moment (LPM) was developed by Bawa (1975) and Fishburn (1977), and it encompasses a significant number of the known Von Neumann-

Morgenstern utility functions. It allows the use of all risk aversion coefficients. Given an investor risk aversion value, a, the LPM is defined as:

$$\mathrm{LPM}(a,t) = \frac{1}{K}\sum_{T=1}^{K} \mathrm{Max}.[0,(t-R_T)]^a \qquad (1)$$

where t is the target return (which can be zero, the mean return or any other threshold considered relevant by the investor), K is the number of observations and R_T is the return for the asset during time period T, $T=1, \ldots, K$. When $a = 1$, the investor is neutral toward risk. The investor is risk-averse if $a > 1$ and risk-loving if $a < 1$. Obviously, the critical parameter of this measure is the parameter a. The LPM values are largely dependent on the degree of skewness in the distribution returns. LPM values are greater when the skewness in the asset return distributions is negative for a risk-averse investor, and are lower for a risk-loving investor. In our opinion, this is the most perfect measure of risk because, like stochastic dominance, it does not make any distributional assumptions and assumes a very general set of utility functions.

2.2. Semi-Standard Deviation

Semi-Standard deviation (SSD) is computed as

$$\mathrm{SSD} = \sqrt{1/K \sum_{T=1}^{K}(R_T - s)^2} \qquad \text{For all } R_T < s \qquad (2)$$

where s is the set value (defined as t in equation 1). When $s = t$ and $a = 2$ the square root of LPM becomes SSD. Several authors, like Harlow (1991), Sortino and Forsey (1996), Estrada (2000) and Plantinga and Groot (2001), focus on SSD as a measure of downside risk and use it in asset allocation, performance measurement or estimation of the cost of equity.

2.3. Leland's Beta

Leland (1999) argues that lognormal distribution is more satisfactory as a description of asset returns than the normal since it has zero probability of negative values and positive skewness. The author says that the average investor must have a power utility function (this utility implies skewness preference) and he develops his work based on the equilibrium equation presented by Rubinstein (1976).

$$P_0 = \frac{E\{(1+R_p)P_0\} - \lambda\rho\{(1+R_p)P_0, -(1+R_M)^{-b}\}\sigma_{\{(1+R_p)P_0\}}}{1+R_f} \tag{3}$$

where, P_0 is the price of an asset; R_p, R_M are the portfolio p and market portfolio M returns, respectively, over a time period; $\rho\ \{x, y\}$ is the correlation coefficient between x and y; E(.) is the expectation operator; and σ is the $\lambda = \sigma_{(1+R_M)^{-b}} / E\{(1+R_M)^{-b}\}$ standard deviation. Since this equation must also hold for the market portfolio, dividing both sides of equation (3) by P_0 and rearranging terms, we get

$$E(R_p) = r_f + B_p\{E(R_M) - r_f\} \tag{4}$$

The term B_p is the modified beta, resulting from:

$$B_p = \frac{\mathrm{cov}_{R_p, -(1+R_M)^{-b}}}{\mathrm{cov}_{R_M, -(1+R_M)^{-b}}} \tag{5}$$

Rubinstein (1976) shows that b can be defined as:

$$b = \frac{\ln\{E(1+R_M)\} - \ln(1+r_f)}{\sigma^2_{\ln(1+R_M)}} \tag{6}$$

and is therefore related to the market excess return when market returns follow a lognormal distribution. If we compare the traditional beta (β) equation with the B_p equation, we find that the risk measures are related to each other. It is interesting to note that the information necessary to compute B_p is basically the same used to compute β.

2.4. Downside CAPM

Estrada (2002) proposes the Downside CAPM (D-CAPM). This is an alternative pricing model for diversified investors, which is based on an alternative measure of risk for diversified investors, the downside beta, βd. The principle behind D-CAPM is the same as for CAPM[2] but the measure of systematic risk, β, is changed to incorporate skewness. The βd is defined as:

[2] Investors' demand for a rate of return that equals the risk free rate plus a risk premium proportional to assets systematic risk.

$$\beta d = \frac{E\,\{Min[(R_i - \mu_i)],0].Min[(R_M - \mu_M), 0]\}}{E\,\{Min[(R_M - \mu_M),0]^2\}} \tag{7}$$

where R and μ represent returns and mean returns, respectively. The Downside beta is a measure of systematic downside risk, thus it is the measure of risk to a diversified investor, who owns assets with asymmetric returns distribution.

3. DATA AND METHODOLOGY

The data used in this study consist of daily returns (where the returns are computed by natural logarithm differences), adjusted for dividends and stock splits, from the Portuguese Stock Index PSI 20 and a sample of stocks traded on this market. PSI 20 is used as the market proxy and we choose a total of 31 stocks, which include those in the PSI 20. Summary statistics of the data about these stocks and PSI 20 are provided in Table 1. Returns used throughout the article are daily returns, ranging from the first transaction day until August 2000.

To apply downside risk models, we need to prove: first, the stocks' return distributions are non-normal and therefore it is not correct to use mean-variance models; second, the stocks' return distributions are skewed.

Table 1 reports the results of statistical normality tests (Studentized Range and the Jarque-Bera statistics[3]). The kurtosis, skewness, Studentized Range and Jarque-Bera statistics indicate, for almost all stocks, a clear rejection of the normality of daily returns[4]. Aparício and Estrada (2001) conclude the same for European securities markets. Only for PT Multimédia is the Jarque-Bera statistic not statistically significant at a 90% level.

Nevertheless, the presence of asymmetry is not yet clearly proven because the rejection of normality does not imply the rejection of symmetry. Thus, it seems important to verify whether there is asymmetry around the mean. As a preliminary analysis, we divided each stock sample period in two sub-periods with equal number of observations[5]. The results suggest that skewness is not time-varying, since the skewness does not change significantly from one period to the other, while other descriptive statistics, such as kurtosis

[3] The Jarque-Bera statistic asymptotically follows a χ^2 distribution with two degrees of freedom.

[4] We also employ BestFit to perform a further robustness check of the previous results under other assumptions. We conclude that, at a 5% confidence level, the normal distribution is clearly rejected relatively to the whole sample. The Kolmogorov-Smirnov and Anderson-Darling ranking tests position most and all of the stocks and the market index in the alternative logistic distribution (which allows for some asymmetry and therefore the downside risk models used). The results are available upon request.

[5] Thanks are due to the referee for this suggestion.

TABLE 1

Summary Statistics (daily euro returns)

	No.	Mean Return (daily)	t - stat.	Mean Return (year)	Standard Deviation	Kurtosis	Skewness	SK stand. error	Studentized range	Jarque-Bera		Start
										Statistic	*p* -value	
BANIF	1899	0.0000	0.0824	0.9%	31.0%	8.561	0.292	0.0562	14.12 **	2474 ***	0.0000	Jan-93
BCP	1578	0.0005	1.3351	12.6%	23.6%	11.721	-0.402	0.0617	15.59 **	5043 ***	0.0000	Apr-94
BES	1899	0.0004	1.2492	10.6%	23.5%	19.114	-0.916	0.0562	18.69 **	20811 ***	0.0000	Jan-93
BPA	1824	0.0004	0.8037	10.5%	35.5%	60.166	4.087	0.0574	21.50 **	253439 ***	0.0000	Feb-93
BPI	1158	0.0007	1.0825	17.2%	34.1%	65.850	-3.850	0.0720	21.91 **	193454 ***	0.0000	Dec-95
BPSM	1286	0.0011	2.1593	27.8%	29.2%	7.373	1.001	0.0683	12.22 **	1239 ***	0.0000	Jun-95
Brisa	685	0.0008	1.4196	20.5%	24.0%	1.974	0.474	0.0936	8.48 **	56 ***	0.0000	Nov-97
BTA	1899	0.0003	0.6044	7.1%	32.6%	67.662	2.905	0.0562	28.04 **	333503 ***	0.0000	Jan-93
C. Amorim	1899	0.0005	0.9968	11.5%	31.8%	5.984	0.505	0.0562	11.81 **	785 ***	0.0000	Jan-93
Cimpor	1522	0.0007	1.7933	16.4%	22.5%	46.636	-2.543	0.0628	22.25 **	122391 ***	0.0000	Jul-94
CIN	1899	0.0006	1.3016	14.5%	30.7%	13.470	-0.263	0.0562	17.08 **	8695 ***	0.0000	Jan-93
CPP	1899	0.0003	0.6047	6.3%	28.9%	12.589	1.121	0.0562	14.42 **	7673 ***	0.0000	Jan-93
EDP	799	0.0002	0.3425	5.2%	27.1%	4.734	0.404	0.0867	11.21 **	122 ***	0.0000	Jun-97
Engil	1339	0.0002	0.3973	5.5%	31.8%	57.473	3.498	0.0669	23.36 **	168282 ***	0.0000	Apr-95
Império	1156	0.0003	0.5554	8.6%	33.4%	10.259	0.709	0.0720	13.93 **	2635 ***	0.0000	Jan-96
M. Confiança	1899	0.0010	1.4592	24.0%	45.3%	28.970	-0.990	0.0562	22.03 **	53675 ***	0.0000	Jan-93
Modelo Continente	1156	0.0009	1.5404	21.8%	30.5%	8.958	-0.322	0.0720	12.42 **	1730 ***	0.0000	Jan-96
Mota & Comp.	1899	0.0000	0.0934	1.1%	33.4%	21.394	-0.932	0.0562	18.31 **	27045 ***	0.0000	Jan-93
Mundicenter	1899	0.0006	1.2781	15.5%	33.5%	8.872	0.405	0.0562	12.60 **	2780 ***	0.0000	Jan-93
Pararede	292	0.0033	1.1699	82.9%	76.6%	18.466	2.839	0.1433	10.43 **	3302 ***	0.0000	Jul-99
Portucel	1282	0.0002	0.4536	5.8%	28.8%	5.352	0.480	0.0684	12.20 **	345 ***	0.0000	Jun-95
PT	1298	0.0011	1.9399	26.9%	31.6%	3.496	-0.270	0.0680	8.90 **	29 ***	0.0000	Jun-95
PT Multimédia	195	0.0005	0.1553	12.9%	73.6%	2.874	-0.246	0.1754	7.45 **	2	0.3503	Nov-99
SAG	530	-0.0003	-0.3835	-8.7%	33.1%	6.184	0.414	0.1064	11.36 **	239 ***	0.0000	Jul-98
Semapa	1261	0.0009	1.8386	22.3%	27.2%	6.546	-0.428	0.0690	12.22 **	699 ***	0.0000	Jul-95
Soares da Costa	1656	-0.0008	-1.3777	-19.3%	36.0%	10.127	0.167	0.0602	16.31 **	3513 ***	0.0000	Dec-93
Sonae Indústria	1673	0.0003	0.4287	6.8%	41.1%	36.000	0.067	0.0599	24.87 **	75914 ***	0.0000	Nov-93
Sonae Investimentos	1899	0.0012	2.5095	29.4%	32.3%	5.674	0.299	0.0562	12.29 **	594 ***	0.0000	Jan-93
Soporcel	1899	0.0009	1.9004	22.4%	32.5%	26.782	2.051	0.0562	19.22 **	46084 ***	0.0000	Jan-93
Telecel	922	0.0012	1.3451	30.9%	44.1%	3.102	0.187	0.0807	9.46 **	5.8 *	0.0556	Dec-96
Unicer	1832	0.0006	1.7494	15.6%	24.1%	22.131	1.783	0.0572	18.53 **	28909 ***	0.0000	Jan-93
PSI 20	1899	0.0007	2.9311	18.3%	17.2%	9.249	-0.755	0.0562	15.17 **	3271 ***	0.0000	Jan-93

Skewness = m_3 / s^3; Kurtosis = m_4 / s^4; Studentized range = [max. (R_t) – min. (R_t)] / s ; and Jarque-Bera = T* (Skewness²/6 + (Kurtosis – 3)²/24); where $m_k = \sum (R_t - \mu)^2 / (T-1)$, T is the number of observations and μ is the mean return. Standard errors of the coefficients of skewness (SK stand. error) under the null hypothesis of normality where computed as $(6 / T)^{1/2}$.
* Confidence level at 10%; ** Confidence level at 5%; *** Confidence level at 1%.

and variance, do. We therefore continue to reject the null hypothesis of normality of daily returns for both sub-periods because the sign of the skewness does not change from the first to the second sub-period. As a result, it seems that downside risk is a relevant measure of risk, independent of the time period considered .

Following the procedure proposed by Peiró (1999) and Machado-Santos and Fernandes (2005), we created two sub-samples for each series[6]. If returns are symmetric (around the mean), these two sub-samples should have the same distribution. To test the hypothesis of significant differences between the two sub-samples, we carry out the parametric F-test and the non-parametric Kruskal-Wallis rank test. The main difference between these tests is the assumption of the nature of the distributions to check for different mean values among various populations. The F-test for K population means is used to test the null hypothesis that the K samples came from K populations with the same mean. The Kruskal-Wallis, which assumes similar distributions among the populations, is used to test the null hypothesis that all K population distribution functions are identical or, alternatively, the K populations have equal means. The results of these tests are shown in Table 2. The results suggest the rejection of null hypothesis of equality between positive and negative excess returns for almost all individual stocks (except for Modelo Continente, PT Multimédia and Telecel) and the acceptance for the market index (PSI 20), at a 5% significant level, by the F-test and Kruskal-Wallis test. It seems that the Portuguese stock skewness is diversifiable. This was already demonstrated by Levy and Markowitz (1979) and Kroll, Levy and Markowitz (1984) for other markets, and supported their thesis that quadratic utility function can be used to maximize investor's utility. Peiró (1999) also finds weak signs of asymmetry in most international market indexes' daily financial returns.

These results support the arguments against the use of CAPM or mean-variance models on the estimation of the cost of equity. On the other hand, the observed asymmetry of stock return distributions favours the use of downside models.

Any required return can be thought of as having two components: a risk free rate and a risk premium. To estimate an investor's required return (or a firm's cost of equity) we apply section 2 downside risk measures in the following equation[7]:

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[6] One of the series is formed by negative excess returns in absolute values, $|R^-| = \{\mu - R_t \mid R_t < \mu\}$, and the other is formed by positive excess returns, $|R^+| = \{\mu - R_t \mid R_t > \mu\}$.

[7] For instance, for LPM (2, μ) equation (8) becomes:

$$K_{i,e} = r_f + (R_M - r_f)\left[\sqrt{\frac{1}{K}\sum_{T=1}^{K} Max.[0,(\mu_i - R_T)]^2} \Big/ \sqrt{\frac{1}{K}\sum_{T=1}^{K} Max.[0,(\mu_M - R_T)]^2}\right]$$

where μ_i and μ_M represent stock *i* and market (proxied by PSI20) mean return, respectively, computed using the same time period.

TABLE 2

F- Test and Kruskall-Wallis (KW) Tests for Differences Between Positive and Negative Returns

	F-stat	P value	F crit. 5%	F crit. 1%	KW-stat	P value	χ^2 crit. 5%	χ^2 crit. 1%
BANIF	146.2528	0.00000	3.8464	6.6483	341.8889	0.000	3.841	6.635
BCP	4.0545	0.04422	3.8474	6.6510	3.9075	0.048	3.841	6.635
BES	43.0401	0.00000	3.8464	6.6483	111.4495	0.000	3.841	6.635
BPA	34.6300	0.00000	3.8466	6.6489	106.0127	0.000	3.841	6.635
BPI	29.2861	0.00000	3.8495	6.6568	6.6606	0.010	3.841	6.635
BPSM	20.4886	0.00001	3.8487	6.6547	14.3091	0.000	3.841	6.635
Brisa	9.2776	0.00241	3.8551	6.6722	5.3329	0.021	3.841	6.635
BTA	10.7240	0.00108	3.8464	6.6483	25.8990	0.000	3.841	6.635
Cimpor	6.2818	0.01228	3.8464	6.6483	16.5641	0.000	3.841	6.635
CIN	144.4689	0.00000	3.8476	6.6516	350.4355	0.000	3.841	6.635
Corticeira Amorim	27.3416	0.00000	3.8464	6.6483	38.5321	0.000	3.841	6.635
CPP	63.3852	0.00000	3.8464	6.6483	162.3672	0.000	3.841	6.635
EDP	11.6395	0.00068	3.8532	6.6668	7.2075	0.007	3.841	6.635
Engil	39.0062	0.00000	3.8484	6.6539	92.6042	0.000	3.841	6.635
Império	30.8117	0.00000	3.8495	6.6569	66.1606	0.000	3.841	6.635
M. Confiança	43.3150	0.00000	3.8464	6.6483	149.7062	0.000	3.841	6.635
Modelo Continente	0.8749	0.34981	3.8495	6.6569	2.7090	0.100	3.841	6.635
Mota & Companhia	185.7268	0.00000	3.8464	6.6483	486.6554	0.000	3.841	6.635
Mundicenter	190.9443	0.00000	3.8464	6.6483	413.4011	0.000	3.841	6.635
Pararede	7.1962	0.00773	3.8737	6.7231	3.6188	0.057	3.841	6.635
Portucel	7.6950	0.00562	3.8487	6.6548	7.9552	0.005	3.841	6.635
PT	3.9684	0.04657	3.8486	6.6544	8.4598	0.004	3.841	6.635
PT Multimédia	0.0839	0.77244	3.8901	6.7680	0.0597	0.807	3.841	6.635
SAG	4.2559	0.03960	3.8591	6.6831	12.2125	0.000	3.841	6.635
Semapa	9.1071	0.00260	3.8489	6.6551	28.7593	0.000	3.841	6.635
Soares da Costa	11.6647	0.00065	3.8471	6.6502	73.8674	0.000	3.841	6.635
Sonae Indústria	20.6888	0.00001	3.8470	6.6501	40.2152	0.000	3.841	6.635
Sonae Investimentos	7.9431	0.00488	3.8464	6.6483	8.5920	0.003	3.841	6.635
Soporcel	101.5479	0.00000	3.8464	6.6483	210.7107	0.000	3.841	6.635
Telecel	3.3604	0.06711	3.8516	6.6625	1.9828	0.159	3.841	6.635
Unicer	30.7136	0.00000	3.8465	6.6487	63.9889	0.000	3.841	6.635
PSI 20	0.3359	0.56227	3.8464	6.6483	2.3853	0.1225	3.841	6.635

$$K_{i,e} = r_f + (R_M - r_f)\,(DRM_i/DRM_M) \tag{8}$$

where $K_{i,e}$ is the cost of equity of firm i, r_f is the risk free rate, R_M is the market rate of return, DRM_i is the downside risk measure of firm , and DRM_M is the downside risk measure of the market for the same period of firm i.

Besides downside risk measures, we also use the traditional beta (from CAPM) and the total risk (TR - measured by standard deviation) in the estimation of a firm's cost of equity. The LPM and SSD are estimated with respect to the mean of stock returns. The risk aversion coefficient of LPM measure, a, is set to 0.5, 1, 2 and 3 (the increase of a means an increase of risk aversion).

In this sense, specific risk measures should be used in particular circumstances:

I. Traditional beta when the investor is diversified and stock and market returns distribution are symmetric;
II. TR when the investor is not diversified and stock and market returns distribution are symmetric;

III. Leland's beta when the investor is diversified but the stock return distributions are skewed and the market return distribution is lognormal;
IV. Downside beta (β_d) when the investor is diversified but the stocks and market return distributions are skewed;
V. LPM and SSD when the investor is not diversified and stock return distribution is skewed.

4. EMPIRICAL EVIDENCE

In this section, we analyse the importance of downside risk models in the estimation of the cost of equity and compare them to standard models. This analysis is done in two ways: first, we regress several risk measures with mean returns to identify the important ones; second, we estimate the companies' cost of equity.

Table 3 presents the estimates of each of the eight risk measures for each of the stocks in the sample, computed over the whole sample period considered for each stock.

The correlation matrix between the mean returns and the risk measures (Table 4) provides a preview of some results of this study. We observe a high correlation between the mean return and LPM 0.5 and TR, which suggests that the investors that hold our sample companies' stocks are both risk-loving and do not hold a diversified portfolio. The high correlation between TR and all LPM measures suggests that an important component of total risk is related to the negative skewness of the stocks' return distributions (see Table 1). The low correlation between the measures of TR/LPM and the measures of systematic risk (Leland's beta, traditional beta and downside beta) contradicts the conclusions of Beedles (1979), who argues for the high correlation between traditional beta and skewness. Rather, it seems to indicate that idiosyncratic risk represents an important fraction of total risk/downside risk. The high correlation between the measures of systematic risk points towards a low level of stocks' systematic skewness and that the return distribution of the market index is symmetric (see Table 2). The low correlation between the mean return and LPM 3 suggests that investors are not risk-averse.

A regression analysis provides a more detailed analysis about the relationship between the risk measures and stock returns. In Table 5, we make a cross-sectional simple linear regression model relating mean returns (MR) to the defined risk variables (RV).

The results shown in Table 5 are corrected for heteroskedasticity. Firstly, it should be noted that all regressions are statistically significant according to

TABLE 3

Risk Measures (daily euro returns)

Stock	A-D Rank	Mean Return (year)	LPM 0.5	LPM 1	LPM 2 (or SDD)	LPM 3	B	βd	β	TR
BANIF	*Logistic*	0.9%	320%	118%	21%	5%	0.48 (32.7)	0.54 (14.1)	0.46 (11.6)	31.0%
BCP	*Logistic*	12.6%	327%	109%	17%	4%	0.81 (73.2)	0.84 (32.8)	0.82 (31.7)	23.6%
BES	*Logistic*	10.6%	319%	104%	17%	4%	0.81 (87.8)	0.86 (33.8)	0.80 (31.9)	23.5%
BPA	*Logistic*	10.5%	339%	118%	21%	5%	0.63 (37.4)	0.62 (16.8)	0.62 (13.6)	35.5%
BPI	*Lognormal*	17.2%	355%	128%	26%	11%	0.27 (14.5)	0.46 (8.7)	0.29 (5.8)	34.1%
BPSM	*Logistic*	27.8%	354%	121%	19%	4%	0.64 (43.4)	0.71 (22.4)	0.64 (16.5)	29.2%
Brisa	*Logistic*	20.5%	354%	118%	16%	3%	0.53 (28.1)	0.50 (17.0)	0.54 (16.1)	24.0%
BTA	*Logistic*	7.1%	344%	121%	21%	6%	0.71 (92.9)	0.66 (18.3)	0.72 (17.9)	32.6%
C. Amorim	*Logistic*	11.5%	360%	128%	21%	5%	0.77 (223.5)	0.83 (24.5)	0.75 (19.3)	31.8%
Cimpor	*Logistic*	16.4%	318%	103%	17%	5%	0.65 (62.0)	0.68 (23.0)	0.65 (23.3)	22.5%
CIN	*Logistic*	14.5%	333%	116%	22%	6%	0.39 (70.8)	0.41 (10.4)	0.39 (9.7)	30.7%
CPP	*Logistic*	6.3%	326%	111%	19%	4%	0.64 (174.3)	0.65 (20.7)	0.64 (17.9)	28.9%
EDP	*Logistic*	5.2%	358%	122%	18%	4%	0.71 (39.6)	0.69 (23.4)	0.72 (21.7)	27.1%
Engil	*Logistic*	5.5%	340%	117%	19%	4%	0.45 (26.4)	0.53 (14.5)	0.44 (9.8)	31.8%
Império	*Logistic*	8.6%	354%	125%	22%	5%	0.77 (46.6)	0.80 (20.7)	0.76 (17.3)	33.4%
M. Confiança	*Logistic*	24.0%	366%	138%	33%	13%	0.86 (49.0)	0.75 (12.6)	0.87 (15.3)	45.3%
Modelo Continente	*Logistic*	21.8%	349%	124%	22%	5%	0.93 (68.7)	1.07 (31.8)	0.91 (25.2)	30.5%
Mota & Comp.	*Logistic*	1.1%	299%	111%	24%	8%	0.35 (42.1)	0.43 (9.5)	0.34 (7.9)	33.4%
Mundicenter	*Logistic*	15.5%	338%	119%	23%	6%	0.17 (37.8)	0.27 (6.5)	0.18 (4.0)	33.5%
Pararede	*Logistic*	82.9%	439%	178%	41%	13%	1.01 (9.6)	0.97 (5.9)	1.00 (4.6)	76.6%
Portucel	*Logistic*	5.8%	356%	125%	19%	4%	0.70 (48.6)	0.71 (21.4)	0.71 (19.0)	28.8%
PT	*Logistic*	26.9%	360%	130%	23%	5%	1.26 (106.4)	1.38 (47.5)	1.27 (42.1)	31.6%
PT Multimédia	*Logistic*	12.9%	450%	202%	52%	17%	1.05 (8.9)	0.86 (3.8)	1.08 (4.9)	73.6%
SAG	*Logistic*	-8.7%	356%	133%	23%	5%	0.65 (8.4)	0.68 (13.9)	0.64 (11.9)	33.1%
Semapa	*Logistic*	22.3%	348%	120%	20%	4%	0.75 (57.5)	0.87 (27.1)	0.72 (21.1)	27.2%
Soares da Costa	*Logistic*	-19.3%	356%	134%	25%	6%	0.46 (23.2)	0.49 (10.9)	0.45 (9.4)	36.0%
Sonae Indústria	*Logistic*	6.8%	370%	137%	28%	10%	1.02 (49.0)	1.08 (23.2)	1.00 (19.6)	41.1%
Sonae Investimentos	*Logistic*	29.4%	362%	130%	22%	5%	1.15 (322.6)	1.19 (38.6)	1.17 (35.0)	32.3%
Soporcel	*Logistic*	22.4%	351%	121%	21%	5%	0.43 (89.2)	0.43 (11.7)	0.43 (10.2)	32.5%
Telecel	*Logistic*	30.9%	400%	157%	30%	7%	1.26 (57.5)	1.34 (29.0)	1.25 (24.5)	44.1%
Unicer	*Logistic*	15.6%	330%	107%	15%	3%	0.40 (28.3)	0.45 (16.9)	0.40 (12.7)	24.1%
PSI 20 (market proxy)	*Logistic*	18.3%	305%	95%	13%	2%	1 -	1 -	1 -	17.2%

A-D Rank: Anderson-Darling Ranking for goodness-of-fit; SSD: Semi-standard deviation (equal to LPM when $a = 2$); B: Leland's Beta; β: Traditional beta (from CAPM); βd: Downside beta; TR: Total Risk.
(t-statistics are provided in parenthesis).

TABLE 4

Cross-Section Analysis: Correlation Matrix

	Mean Return	LPM 0.5	LPM 1	LPM 2	LPM 3	B	β	βd	TR
Mean Return	1								
LPM 0.5	0.52	1							
LPM 1	0.40	0.96	1						
LPM 2	0.36	0.83	0.94	1					
LPM 3	0.32	0.68	0.80	0.93	1				
B	0.41	0.57	0.52	0.40	0.25	1			
β	0.41	0.58	0.53	0.41	0.27	1.00	1		
βd	0.40	0.46	0.40	0.27	0.25	0.96	0.96	1	
TR	0.49	0.86	0.93	0.96	0.87	0.39	0.40	0.39	1

B: Leland's Beta; β: Traditional beta (from CAPM); βd: Downside beta; TR: Total Risk.

the F statistic and that LPM 3 is the risk measure with the lowest explanatory power, which reinforces the idea that investors who hold our sample stocks are not risk-averse. All risk measures enter the regressions with the expected signs. Only the measures of systematic risk (B, β, βd) are statistically significant, at the 1 percent confidence level, but their explanatory power is relatively low. On the other hand, the measures of total risk and LPM are not statistically significant, at the 1 percent confidence level, but their explanatory power is relatively high. Regressions with LPM 0.5 and TR have high explanatory power. Estrada (2000) reports similar results. The author finds that TR is significantly related to stock returns, which, combined with the lack of explanatory power of systematic risk, implies that in emerging markets diversifiable risk is priced.

Table 6 reports the results, corrected for heteroskedasticity, of multiple regressions in which returns are jointly related with two risk variables[8]. These results differ only slightly from the previous ones. LPM measures, when investor is risk neutral and averse (a =1, 2, 3), remain not statistically significant, with low explanatory power. The measures of systematic skewness and systematic risk, when combined with the measures of total risk or LPM, remain statistically significant and the explanatory power of the regressors increase. This means that these measures seem to explain different parts of the variability of the mean returns. It is worth noting that when one of the risk variables of the regression is TR, regression explanatory power increases significantly. This means that TR explains part of the variability in mean returns not considered by the other measures.

We estimate the cost of equity of the companies using all risk measures. As stated previously, the right model to estimate the cost of equity is a function of the investor's risk exposure and of the return distribution features of owned asset(s).

TABLE 5

Cross-Section Analysis: Simple Regressions

$MR_i = a + b * RV_i + u_i$						
RV	a	t-stat	b	t-stat	F	Adj. R^2
LPM 0.5	-0.74	-1.52	0.25	1.76	9.52***	0.215
LPM 1	-0.22	-0.77	0.30	1.21	4.98**	0.114
LPM 2	-0.016	-0.13	0.74	1.15	3.94**	0.086
LPM 3	0.06	1.01	1.49	1.22	2.97*	0.060
B	-0.02	-0.33	0.25	2.41	5.83**	0.138
β	-0.02	-0.34	0.25	2.48	5.93**	0.141
βd	-0.03	-0.48	0.24	2.77	5.56**	0.132
TR	-0.06	-0.47	0.63	1.51	8.52***	0.195

MR: Mean Return; RV: Risk Variable; B: Leland's Beta; β: Traditional beta (from CAPM); βd: Downside beta; TR: Total Risk. * Confidence level at 10%; ** Confidence level at 5%; *** Confidence level at 1%.

[8] From the analysis of Table 4 we can conclude that we will have problems of multicollinearity if we relate returns with two perfect or almost perfect correlated risk variables. As Gujarati (1988) says *"...we do not have one unique method of detecting or measuring the strength of multicollinearity"* (p. 298), so we consider that when the correlation coefficients between the regressors is in excess of 0.8, multicollinearity could be a serious problem and we do not combine these regressors.

TABLE 6

Cross-Section Analysis: Multiple Regressions

$MR_i = a + b_1 * RV_{1,i} + b_2 * RV_{2,i} + v_i$								
RV_1 / RV_2	a	t-stat	b_1	t-stat	b_2	t-stat	F	Adj. R^2
LPM 0.5 / LPM 3	-0.898	-1.865	0.303	2.206	-0.356	-0.424	5.22**	0.22
LPM 0.5 / B	-0.719	-1.169	0.227	1.187	0.101	1.125	5.67***	0.237
LPM 0.5 / β	-0.716	-1.148	0.226	1.164	0.100	1.087	5.65***	0.237
LPM 0.5 / βd	-0.743	-1.275	0.227	1.255	0.125	1.493	6.07***	0.253
LPM 1 / B	-0.233	-0.734	0.211	0.738	0.165	1.942	3.86**	0.16
LPM 1 / β	-0.229	-0.714	0.208	0.714	0.166	1.941	3.87**	0.16
LPM 1 / βd	-0.274	-0.907	0.234	0.864	0.174	2.241	4.17**	0.174
LPM 2 / B	-0.102	-0.681	0.516	0.740	0.190	2.718	3.79**	0.157
LPM 2 / β	-0.100	-0.668	0.507	0.717	0.190	2.792	3.79**	0.157
LPM 2 / βd	-0.134	-0.902	0.596	0.881	0.199	2.982	4.12**	0.172
LPM 3 / B	-0.068	-0.661	1.153	0.883	0.211	2.890	3.85**	0.16
LPM 3 / β	-0.067	-0.653	1.127	0.855	0.211	3.005	3.85**	0.16
LPM 3 / βd	-0.095	-0.900	1.344	1.029	0.222	3.092	4.17**	0.174
B / TR	-0.143	-0.982	0.154	2.216	0.542	1.136	6***	0.25
β / TR	-0.142	-0.970	0.154	2.218	0.539	1.119	6.01***	0.25
βd / TR	-0.178	-1.230	0.179	2.741	0.574	1.262	6.71***	0.276

MR: Mean Return; RV: Risk Variable; B: Leland's Beta; β: Traditional beta (from CAPM); βd: Downside beta; TR: Total Risk. * Confidence level at 10%; ** Confidence level at 5%; *** Confidence level at 1%.

Table 7 shows the Portuguese cost of equity based on each of the risk measures considered, as well as on a risk free rate of 4% and a market risk premium of 5.5%[9].

As expected, the cost of equity based on measures of systematic risk or systematic skewness is the lowest of all, and the values are identical, which, once again, reiterates the inexistence of systematic skewness and the symmetry of market index. Investors should use these measures when they have a diversified portfolio.

By contrast, we have the cost of equity based on TR. As expected, the cost of equity based on this measure is one of the highest (only when the investor becomes more risk averse, $a > 2$, do we find even higher values). Investors should use this measure when they have a non-diversified portfolio and asset and market return distributions are symmetric. The problem with this measure is that standard deviation gives the same weight to downside and upside risk, but only the downside risk is relevant. Thus, when we have a symmetric market return distribution and we are estimating the cost of equity of an asset with a right (left) skewed return distribution, the TR overestimates (underestimates) this value. For these reasons we disagree with Damodaran (2001) when he says that 'the owner of a private firm generally has the bulk of

[9] Damodaran (2000) says that the particular period, the risk free asset, the periodicity of the quotations, etc, condition the market risk premium. Therefore, we assume that 5.5% is a fair market risk premium for the Portuguese market, which is usually assumed by several academics.

TABLE 7

Portuguese Firms Cost of Equity

	LPM				B	βd	β	TR
	0.5	1	2 (or SSD)	3				
BANIF	9.77%	10.83%	12.88%	17.75%	6.63%	6.96%	6.55%	13.90%
BCP	9.80%	10.18%	11.19%	11.33%	8.48%	8.60%	8.49%	11.20%
BES	9.75%	10.02%	11.19%	15.00%	8.47%	8.71%	8.43%	11.50%
BPA	10.11%	10.83%	12.88%	17.75%	7.44%	7.42%	7.40%	15.24%
BPI	10.24%	10.90%	13.53%	24.17%	5.48%	6.54%	5.58%	13.39%
BPSM	10.28%	10.72%	11.46%	11.33%	7.55%	7.92%	7.52%	12.41%
Brisa	9.69%	9.59%	9.18%	9.50%	6.90%	6.73%	6.96%	9.65%
BTA	10.20%	11.01%	12.88%	20.50%	7.91%	7.63%	7.95%	14.40%
C. Amorim	10.47%	11.33%	12.88%	13.17%	8.21%	8.58%	8.11%	13.78%
Cimpor	9.73%	9.96%	11.19%	17.75%	7.59%	7.74%	7.55%	11.18%
CIN	10.00%	10.72%	13.31%	20.50%	6.15%	6.26%	6.14%	13.80%
CPP	9.88%	10.43%	12.04%	15.00%	7.52%	7.59%	7.50%	13.22%
EDP	9.84%	9.83%	9.82%	11.33%	7.89%	7.78%	7.93%	10.46%
Engil	10.03%	10.57%	11.46%	11.33%	6.48%	6.89%	6.41%	13.34%
Império	10.22%	10.74%	12.07%	13.17%	8.24%	8.40%	8.17%	13.19%
M. Confiança	10.60%	11.99%	17.96%	39.75%	8.75%	8.11%	8.80%	18.46%
Modelo Continente	10.13%	10.69%	12.07%	13.17%	9.11%	9.89%	9.00%	12.38%
Mota & Comp.	9.39%	10.43%	14.15%	26.00%	5.94%	6.36%	5.89%	14.65%
Mundicenter	10.10%	10.89%	13.73%	20.50%	4.94%	5.49%	4.99%	14.69%
Pararede	11.21%	12.98%	20.11%	39.75%	9.53%	9.32%	9.52%	25.30%
Portucel	10.30%	10.94%	11.46%	11.33%	7.84%	7.90%	7.89%	12.28%
PT	10.39%	11.22%	13.04%	13.17%	10.93%	11.58%	10.96%	13.16%
PT Multimédia	11.13%	13.34%	20.82%	35.17%	9.78%	8.72%	9.92%	22.01%
SAG	9.55%	10.10%	11.03%	10.88%	7.58%	7.75%	7.55%	11.74%
Semapa	10.15%	10.60%	11.86%	11.33%	8.12%	8.79%	7.98%	11.78%
Soares da Costa	10.34%	11.60%	14.58%	15.00%	6.52%	6.67%	6.50%	15.06%
Sonae Indústria	10.61%	11.85%	15.85%	22.33%	9.61%	9.94%	9.50%	16.69%
Sonae Investimentos	10.53%	11.53%	13.31%	17.75%	10.33%	10.57%	10.45%	14.30%
Soporcel	10.33%	11.01%	12.88%	17.75%	6.37%	6.34%	6.36%	14.38%
Telecel	10.73%	11.85%	14.31%	16.83%	10.93%	11.37%	10.90%	14.98%
Unicer	9.95%	10.26%	10.35%	12.25%	6.22%	6.47%	6.19%	11.67%
Cost of Equity Avg.	**10.18%**	**10.93%**	**13.08%**	**17.82%**	**7.85%**	**8.03%**	**7.84%**	**14.01%**

SSD: Semi-standard deviation (equal to LPM when *a* = 2); B: Leland's Beta; β: Traditional beta (from CAPM); βd: Downside beta; TR: Total Risk.

his or her wealth invested in the business. Consequently, he or she cares about the total risk in the business rather than just the market risk' (p. 86).

In our opinion, the best measure of risk for the owner of a private or public firm or an entrepreneur is a downside risk measure, except if asset and market return distributions are symmetric.

The cost of equity based on downside risk, which takes into account only the volatility that investors seek to avoid is, as expected, between the cost of equity based on systematic risk or systematic skewness and TR. Not surprisingly, when we increase the value of *a* (i.e. the investor becomes more risk averse), the average cost of equity increases to a value higher than that given by TR.

Given the fact that LPM 2 (or semi-standard deviation) is the only measure of all downside risk measures perceived favourably by Markowitz (1952, 1991) and that Estrada (2004) demonstrates that mean-semi standard deviation behavior is an approximately-correct criterion to maximize the expected utility function, in our opinion this is the right risk measure for the average non-diversified investor. Nevertheless, if an investor has a well defined pattern of risk tolerance, he could use one of the other measures.

5. CONCLUSIONS

There are loads of data proving that stock return distributions are not always normal and are often skewed. Given that investors dislike downside risk, i.e., they price and have a preference positive skewness, asset pricing models must consider other moments, like skewness, and not only mean and variance. We started by describing some downside risk measures and testing the hypothesis of normality and symmetry of the most important Portuguese stocks and of the market index. We concluded that none of the sample asset return distributions was normal and that almost all were skewed (although the market index return distribution was symmetric, which means that the asymmetry of the stocks' returns was diversifiable). These facts sustain the applicability of downside risk measures. We then used these measures as explanatory variables and analysed their explanatory power of mean returns. TR was significantly related to stock returns and the regression explanatory power increased when TR was one of the variables, which means that diversifiable risk is priced. The systematic skewness measures (B and βd) and systematic risk measure (β) were very similar. In fact, the empirical evidence presented in this paper did not allow us to draw conclusions about the usefulness of systematic skewness measures, because the market index return distribution was symmetric. Still, in our opinion, these measures are very useful for the estimation of required return when the investor is diversified and market index return distribution is asymmetric. When the investor is not diversified, it is the choice of risk measure that becomes important.

Finally, it is difficult to say what is the right measure to use on the estimation of cost of equity and if these values reflect the real risk taken on by investors. However, empirical evidence (presented both here and in several other papers) does seem to support the use of downside risk measures more than the traditional measure of systematic risk.

Furthermore, it is obvious that the differences in the required return (based on the several risk measures) are relevant and could have an important impact on company value and even explain why investors value the same asset differently. As a suggestion for future research, we could apply these measures in the evaluation of the performance of portfolios with options and model the evaluation like a dynamic model. Besides this, we also think that it will be very important to develop asset pricing models that consider other higher-order moments, like kurtosis, and extreme events, like bankruptcy.

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Resumo

Os modelos tradicionais de avaliação de activos têm como principal pressuposto o conceito de que a curva de utilidade dos investidores é completamente definida pela média e pela variância da distribuição de rendibilidade dos activos. Este pressuposto requer a normalidade ou a simetria destas distribuições de rendibilidade. Na prática, este pressuposto raramente se verifica e diversos estudos demonstram que os investidores têm em consideração os momentos mais elevados das distribuições de rendibilidades. Neste artigo, analisamos a normalidade e a simetria das distribuições de rendibilidade de uma amostra de empresas portuguesas e estimamos o seu custo do capital próprio utilizando um conjunto de medidas de downside risk.

JEL: G12, G31

Palavras-chave: Distribuições de Rendibilidade Assimétricas; Custo do Capital Próprio; Modelos de *Downside Risk*.