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ASSESSING EFFICIENCY OF PORTUGUESE UNIVERSITIES THROUGH PARAMETRIC AND NON-PARAMETRIC METHODS

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Abstract

In this paper we apply two methodologies to the measurement of efficiency in Portuguese Universities. These are a stochastic frontier parametric method and a non-parametric DEA method. We produce results from both methods in terms of efficiency estimates for each university over the period 2003 to 2005. We also produce, in both methods, unitary cost estimates of teaching students (undergraduates, master, and doctoral students) and of producing research outputs. Results from both methods can be said to be coincident to a certain extent, especially in what concerns the identification of worst performers. The productivity change of Portuguese universities in the period 2003-2005 was also explored using Malmquist indices.

JEL Classification: C23, C61, D24, I20, I23

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1. INTRODUCTION

Modern discussions about the evaluation of efficiency characteristically begin with the work of Farrell (1957), whose ideas fathered two very distinct approaches. The analysis of ratios was extended in the late 1970s by Charnes *et al.* (1978) to include multiple input and multiple output scenarios within a framework that did

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not impose a constant set of weights or parameters on all units of observation. This approach, known as data envelopment analysis (DEA), relies on the methods of linear programming, and has come to be used extensively within the fields of operational research and management science. At about the same time, economists extended the classical regression model in a manner that allows the estimation of cost or production functions where many observations lie within a frontier (Aigner *et al.*, 1977). This statistical method, known as stochastic frontier analysis, allows the efficiency of each unit to be evaluated, and does so with the benefit of the tools of statistical inference, but it achieves this at the cost of imposing parameter restrictions.

Despite their common parentage, DEA and stochastic frontier analysis have developed largely independently of one another. Recent innovations, involving the use of panel data, have however allowed the advantages of both methods to be realised simultaneously, allowing the estimation of a cost (or production) function that does not impose identical weights on all units of observation, yet still retaining the advantages of a statistical framework. The new developments also allow detailed examination of changes in efficiency over time.

Higher education systems have provided a popular experimentation laboratory for these types of methodological development. This is so partly because data on higher education institutions are readily available in many countries, and partly because the efficiency of universities is a matter of legitimate policy concern. Moreover, the multiproduct nature of universities – providing as they do both teaching, technology transfer and research in a wide variety of subject areas– adds a layer of both complication and interest, since the technology adopted by multiproduct organisations characteristically involves refined issues of scale and scope.

Issues surrounding costs and efficiency in Portuguese higher education are of particular policy importance in the current climate. Portugal lags behind almost all other OECD countries in terms of the proportion of its population experiencing higher education (OECD, 2006). The desire to expand participation in higher education requires an assurance that such expansion can be achieved in a cost-effective manner. The present paper seeks to contribute to the debate by evaluating cost structures and efficiency within the higher education system in Portugal, using both stochastic frontier and DEA methods.

In other countries, such methods have been used extensively over recent years. The early work on multiproduct cost functions for universities (Cohn *et al.*, 1989) did not use frontier methods, but subsequent studies in the years that followed did (Johnes, 1996; Izadi *et al.*, 2002; Johnes *et al.*, 2005). The most recent studies for England (Johnes and Johnes, 2006), Italy (Agasisti and Johnes, 2006), Spain (Johnes and Salas-Velasco, 2006) and Germany (Johnes and Schwarzenberger, 2006) all employ random parameters stochastic frontier methods.

Our aims in the present paper are to extend such analysis to the case of Portugal, and also to employ the most up-to-date DEA methods to Portuguese data.

The structure of the remainder of the paper is as follows. In Section 2 we introduce the methodology. Data issues are discussed in Section 3. The following section presents the results. We finish with a conclusion and some suggestions for further research.

2. METHODOLOGY

Two broad approaches will be adopted in the empirical analysis reported in this paper. The first is a random parameters stochastic frontier approach. The second uses a non-parametric technique, data envelopment analysis (DEA) as a means of obtaining Malmquist indices as measures of efficiency change over time. These methods of analysis are discussed further below.

2.1 Random parameters stochastic frontier models

Aigner *et al.* (1977) pioneered the stochastic frontier approach to estimating cost and production envelopes. Following this method, the cost function equation

$$y_i = \alpha + \boldsymbol{\beta}'\mathbf{x}_i + v_i + u_i \tag{1}$$

is estimated using maximum likelihood, where v_i denotes normally distributed white noise error and u_i is a second residual term that is intended to capture efficiency differences across observations. This could in principle follow any distribution other than the normal, so that it can be distinguished from the white noise component. In practice, the half-normal is a common assumption, and this is what we assume in the empirical work that follows.

Jondrow *et al.* (1982) showed that it is possible to recover observation-specific estimates of the efficiency residual as

$$\mathrm{E}[u_i|\varepsilon_i] = \sigma\lambda\{\phi(a_i)/[1-\Phi(a_i)] - a_i\}/(1+\lambda^2) \tag{2}$$

where $\sigma = (\sigma_v^2 + \sigma_u^2)^{1/2}$, $\lambda = \sigma_u / \sigma_v$, $a_i = \pm\varepsilon_i\lambda / \sigma$, and $\phi(.)$ and $\Phi(.)$ are, respectively, the density and distribution of the standard normal. The ability to recover such efficiency estimates makes the stochastic frontier approach particularly appealing. Like DEA, it allows efficiency to be evaluated, but it does so in a parametric manner that opens up the toolkit of statistical inference.

The traditional parametric approach has limitations, of course, in that it imposes a common set of (input and) output weights on all decision-making units. However, recent innovations by Tsionas (2002) and Greene (2005) mitigate this shortcoming by drawing on the power of panel data analysis. When using panel data, (1) may be modified to

$$y_{it} = \alpha_i + \boldsymbol{\beta}'_i \mathbf{x}_{it} + v_{it} + u_{it} \tag{3}$$

where $v_{it} \sim N\left[0,\sigma_v^2\right]$, $u_{it} = |U_{it}|$, $U_{it} \sim N\left[0,\sigma_{ui}^2\right]$, and v_{it} is independent of u_{it}. Note that this equation now allows the coefficients, $\boldsymbol{\beta}$, to be institution-specific. Equation (2) is likewise modified, for the panel data case, to

$$E[u_{it}|\varepsilon_{it}] = \sigma\lambda\{\phi(a_{it})/[1-\Phi(a_{it})] - a_{it}\}/(1+\lambda^2) \tag{4}$$

In the present paper, we model the $\boldsymbol{\beta}_i$ as random parameters. Greene (2005) summarises the problem by defining the stochastic frontier as (3) above, the inefficiency distribution as a half-normal with mean $\mu_i = \boldsymbol{\mu}'_i\mathbf{z}_i$ and standard deviation $\sigma_{ui} = \sigma_u \exp(\theta'_i\mathbf{h}_i)$. The parameter heterogeneity may then be modelled as follows:

$$\left.\begin{aligned}
(\alpha_i, \boldsymbol{\beta}_i) &= (\bar{\alpha}, \bar{\boldsymbol{\beta}}) + \Delta_{\alpha,\beta}\mathbf{q}_i + \boldsymbol{\Gamma}_{\alpha,\beta}\mathbf{w}_{\alpha,\beta_i} \\
\boldsymbol{\mu}_i &= \bar{\boldsymbol{\mu}} + \boldsymbol{\Delta}_{\mu}\mathbf{q}_i + \boldsymbol{\Gamma}_{\mu}\mathbf{w}_{\mu i} \\
\boldsymbol{\theta}_i &= \bar{\boldsymbol{\theta}} + \boldsymbol{\Delta}_{\theta}\mathbf{q}_i + \boldsymbol{\Gamma}_{\theta}\mathbf{w}_{\theta_i}
\end{aligned}\right\} \tag{5}$$

The random variation appears in the random parameters vector $\mathbf{w}_{ji}$ (where i is the index of producers and j refers to either the constant, the slope parameter, or – in more general specifications of the model - the moments of the inefficiency distribution represented by $\boldsymbol{\mu}$ and $\boldsymbol{\theta}$); this vector is assumed to have mean vector zero and, in the case where parameters are assumed to be normally distributed, the covariance matrix equals the identity matrix. The $\mathbf{q}_i$ is, in our application, a vector of ones and the $\boldsymbol{\Delta}$ and Γ terms are matrices of parameters that are to be estimated. At an intuitive level, the first line of (5) tells us that the parameters of the cost function are each determined as a constant plus (in a general model) a weighted sum of the determinants of these parameters (included in the vector $\mathbf{q}$) plus an institution-specific shifter given by the $\boldsymbol{\Gamma}\mathbf{w}$ term. This last term is divided into two components; the w are assumed to have a mean of vector zero and a diagonal covariance matrix; the $\boldsymbol{\Gamma}$ is a lower triangular matrix that allows the random parameters to have an unrestricted covariance matrix. The second and third lines of (5) respectively provide analogous expressions for the mean and variance of the inefficiencies.

It is not possible to estimate the parameters of this model by traditional maximum likelihood methods because the unconditional log likelihood includes within it a term containing an unclosed integral. Instead, we adopt a Monte Carlo method in order to maximise the simulated log likelihood function

$$\log L_S = \sum_{i=1}^{N} \frac{1}{R} \sum_{r=1}^{R} \{ \sum_{t=1}^{T} \ln \Phi [[\mu_{ir} / (\sigma_{uir} / \sigma_v) \pm (y_{it} - \alpha_{ir} - \boldsymbol{\beta}'_{ir} \mathbf{x}_{it})(\sigma_{uir} / \sigma_v)] / \sqrt{\sigma_{uir}^2 + \sigma_v^2} \} -$$

$$\frac{1}{2} \{ [\mu_i \pm (y_{it} - \alpha_{ir} - \beta'_{ir} \mathbf{x}_{it})] / \sqrt{\sigma_{uir}^2 + \sigma_v^2} \}^2 + \ln \frac{1}{\sqrt{2\pi}} - \ln \Phi(\mu_i / \sigma_{uir}) - \ln \sqrt{\sigma_{uir}^2 + \sigma_v^2} \} \quad (6)$$

Contemporary software such as Limdep allows this problem to solved relatively easily.

2.2 Functional form of the cost equation

Contemporary studies of costs in higher education institutions originate with the work of Cohn *et al.* (1989), which in turn drew upon the methodological contributions made by Baumol *et al.* (1982). The latter emphasised the nature of costs in a multiproduct setting – and universities provide classic examples of multiproduct organisations, concerned as they are with the production of teaching and research in a range of subject areas. The insight of Baumol *et al.* is that any cost function that is estimated for a multiproduct organisation should, ideally, not *presuppose* the existence of economies of scale or of scope, but that it should allow returns to scale and scope to be estimated. This being so, the functional form of the cost equation estimated for universities in the recent literature has typically followed the recommendations of Baumol *et al.*, and so has been quadratic, hybrid translog, or 'constant elasticity of substitution'.

In the present study, we do not, unfortunately, have the luxury of estimating such refined models. The number of universities in our dataset is small, with just 13 institutions, and the data are not available in consistent form over a long run of years. We therefore simplify the model by estimating a linear specification of the cost function. Given a positive estimate of the intercept term, this has the effect of presuming the existence of scale and scope economies. This is, of course, innocuous inasmuch as it is consistent with the stylised fact that all universities produce teaching at all levels and also produce research. It is not consistent, however, with the fact that more than one university exists – the existence of multiple universities surely has something to do with the fact that returns to scale (eventually) turn negative. However, it is reasonable for us, in what is first and foremost an illustrative exercise, to suppose that the linear specification provides a good approximation to the way in which a (truly nonlinear) cost function behaves locally in the area of the output space that is observed in contemporary Portugal. We should note, however, that for small universities (such as Açores or Madeira) and large universities (such as Porto) in particular, the linear specification may fail to capture the true nature of the technology underpinning costs.

2.3 DEA

DEA is a linear programming non-parametric technique for measuring the relative efficiency of a fairly homogeneous set of decision making units (DMUs) in their use of multiple inputs to produce multiple outputs. It identifies a subset of efficient "best practice" DMUs and for the remaining DMUs, the magnitude of their inefficiency is derived by comparison to a frontier constructed from the "best practices". DEA derives a single summary measure of efficiency for each DMU. For the inefficient DMUs, DEA derives efficient input and output targets and a reference set (or peer group), corresponding to the subset of efficient DMUs to which they were directly compared. Based on Farrell (1957) work, the DEA model was operationalised and popularised by Charnes, Cooper and Rhodes (1978). The input oriented linear programming version of the DEA model introduced by Charnes *et al* (1978) can be formulated as follows:

$$\text{Min} \quad \theta_{j_0}^{C} \tag{7}$$

subject to

$$x_{ij_0}\theta_{j_0}^{C} - \sum_{j=1}^{n} x_{ij}\lambda_j \geq 0, \qquad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{rj_0}, \qquad r = 1, \ldots, s,$$

$$\lambda_j \geq 0, \qquad \forall j$$

where j_0 is the DMU being evaluated in the set of j = 1,...,n DMUs and x_{ij} and y_{rj} denote the observed level of the ith input and rth output at DMU j. Note that in the cost function defined in (1) and (3), *y* stands for the dependent variable (cost) and **x** stands for the vector of independent variables (outputs), which differs from the standard DEA notation of model (7). The value of $\theta_{j_0}^{C}$ is a measure of the technical efficiency (TE) of DMU_{j_0}, which assumes the existence of constant returns to scale (CRS). However, some of the inefficiency detected using this model may be attributable to scale effects, which occur when the observed DMU does not operate at the optimum scale size (operating at variable returns to scale (VRS)). Banker *et al* (1984) extended the original DEA model (7) to account for the existence of variable returns to scale. The VRS model can be obtained through the addition of a convexity constraint to model (7) requiring that the multipliers λ_j add up to 1. The scale efficiency (SE) of a DMU is obtained as the ratio of its technical efficiency (TE, assuming CRS) to its pure technical efficiency (PTE, assuming VRS).

Model (7) is often called the envelopment formulation of the DEA model. By duality, model (8) is an equivalent model, usually referred as the multiplier formulation.

$$\text{Max } e_{j_0} = \sum_{r=1}^{s} u_r y_{rj_0} \tag{8}$$

$$\text{s.t. } \sum_{i=1}^{m} v_i x_{ij_0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad j=1,\ldots,n$$

$$u_r \geq 0, \qquad r=1,\ldots,s$$

$$v_i \geq 0, \qquad i=1,\ldots,m$$

The weights in the multiplier model do not have a clear interpretation. However, in cases where a single input is used it is possible to attribute some economic meaning to the weights. In particular, when the single input is a cost measure (as it is the case in our universities assessment) the output weights can be related to the amount of cost the DMU may be deemed to consume per unit of output (see Dyson and Thanassoulis, 1988).

To see this consider the formulation in (9) which is equivalent to that in (8) except that we assume a single input and a VRS formulation (which implies the addition of a constant to the objective function and restrictions). Note that the normalisation constraint has been removed since for a single input this constraint implies simply that $v = 1 / x_{j0}$.

$$\text{Max } e_{j_0} = \sum_{r=1}^{s} u_r y_{rj_0} + w \tag{9}$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_r y_{rj} - \frac{x_j}{x_{j0}} + w \leq 0, \qquad j=1,...,n$$

$$u_r \geq 0, \qquad r=1,...,s$$

w is free

If instead we normalise the input weight v to be equal to 1 the model would be as shown in (10).

$$\text{Max} \quad e_{j_0} = \sum_{r=1}^{s} u_r y_{rj_0} + w \tag{10}$$

$$\text{s.t.} \quad x_j \geq \sum_{r=1}^{s} u_r y_{rj} + w, \qquad j=1,...,n$$

$$u_r \geq 0, \qquad r=1,...,s$$

w is free

Obviously in this case the value of e_{j_0} would not be bounded by 100%, but efficiency could be recovered from (10) by dividing e_{j_0} / x_{j_0}. Note that efficiency evaluation with a single cost input is a measure of cost efficiency that divides the optimum cost DMU $_{j0}$ should have if efficient by its observed cost.

Using this model we can understand how the universities under evaluation value in monetary terms each of its outputs and how these contribute to observed costs. The reason why we are considering a VRS model is because we believe part of total cost cannot be explained by the variables used (number of undergraduate students, number of post graduate students and publications) and therefore that part is accounted for by a constant term that can be interpreted as a fixed cost. If we interpreted the constant w in (10) as a fixed cost, then we cannot allow it to be negative. In terms of the DEA analysis, adding the non-negative constant to the model implies imposing non-decreasing returns to scale to university activity.

Since in some cases some units will not value at all a given output we can restrict u_r to be above certain levels (as Dyson and Thanassoulis, 1988, also did). Our proposal is to use as lower bound for the weights corresponding to the cost associated with teaching a student at the undergraduate, master or doctoral levels, a value of 2.000 euros. In order to obtain realistic cost estimates, we also defined an upper bound for the weights, with a value of 20.000 euros.

These cost estimates were based on the average cost per year of a master student at the school of engineering of University of Porto, assuming a two-year course with 50% of students attending classes (1st year students) and 50% preparing the research project (2nd year students). The lower bound of the cost estimates is 1/3 below the average cost per year of a master student, assuming a course with 70 students, and the upper bound of the cost estimate is 1/3 above the average cost per student assuming a course with 15 students. Note that according to the legislation concerning the financing of public universities in Portugal[1], engineering students are approximately in a middle position between the most expensive students – medicine, and the least expensive students – law, humanities and social sciences.

We used similar bounds for teaching students in the three levels (undergraduate, master and doctoral) to avoid defining a-priory the students that are more demanding in cost terms. This allowed more flexibility in the DEA analysis to define the university-specific cost estimates. We used master students as basis for defining the cost estimates because in terms of resource requirements, they are a mix between undergraduate students (mostly attending classes) and doctoral students (mostly requiring project supervision).

Since we did not have reliable estimates for the cost of publications, we decided to use similar weight restrictions, as it may be reasonable to assume that a publication should at least require the same resources as teaching a student.

Let k_r and z_r be the agreed minimum and maximum resource input per unit of output, respectively. Then model (10) could be modified as follows for estimating the unitary amount of resource (euros) required by each output, assuming non-decreasing returns to scale.

$$\text{Max} \quad e_{j_0} = \sum_{r=1}^{s} u_r y_{rj_0} + w \qquad (11)$$

$$\text{s.t.} \quad x_j \geq \sum_{r=1}^{s} u_r y_{rj} + w, \qquad j=1,\dots,n$$

$$u_r \geq k_r, \qquad r=1,\dots,s$$

$$u_r \leq z_r, \qquad r=1,\dots,s$$

$$w \geq 0$$

[1] Portaria n.º 231/2006 (Diário da Republica, 2.ª série)

This model gives us an estimate of how each university values its outputs in terms of contribution to total cost.

2.3 Malmquist index

In recent years the Malmquist index has become the standard approach to productivity measurement within the non-parametric literature. Malmquist indexes were introduced by Caves *et al.* (1982). They named these indexes after Malmquist (1953) that proposed constructing input quantity indexes as ratios of distance functions.

To introduce the concept of a distance function, consider that in time period *t* the DMUs are using inputs $X^t \in \Re_+^m$ to produce outputs $Y^t \in \Re_+^s$. The technology of production Φ^t can be defined as follows:

$$\Phi^t = \{(X^t, Y^t) | \textit{Input} \text{ vector } X^t \text{ can produce the output vector } Y^t\}. \tag{12}$$

It consists of all input-output vectors that are technically feasible for a certain production process.

The input distance function is defined on the technology Φ^t as the maximal feasible contraction of X^t that still enables producing Y^t, as follows:

$$D_i(X^t, Y^t) = \sup\left\{\lambda : (\frac{X^t}{\lambda}, Y^t) \in \Phi^t\right\}. \tag{13}$$

The input distance function is the reciprocal to Farrell's input-oriented measure of technical efficiency shown in (7). Note that $D_i(X^t, Yt) \geq 1$ if and only if $(X^t, Y^t) \in \Phi^t$.

To define a Malmquist index requires specification of two mixed-period distance functions, such as:

$$D_i^t(X^{t+1}, Y^{t+1}) = \sup\left\{\lambda : (\frac{X^{t+1}}{\lambda}, Y^{t+1}) \in \Phi^t\right\}; \tag{14}$$

$$D_i^{t+1}(X^t, Y^t) = \sup\left\{\lambda : (\frac{X^t}{\lambda}, Y^t) \in \Phi^{t+1}\right\}. \tag{15}$$

The first mixed-period distance function measures the maximal proportional reduction to inputs required to make a DMU in time period *t+1*, i.e. with (X^{t+1}, Y^{t+1}), efficient in relation to technology at the previous period *t*, i.e. in Φ^t.

Similarly, the second mixed-period distance function measures the maximal proportional reduction to inputs required to make a DMU in time period *t*, i.e.

(X^t, Y^t), efficient in relation to technology at $t+1$, i.e. in Φ^{t+1}. However, since an input-output combination observed in one period may not be feasible within the technology in another period, in both these mixed-period assessments the value of the input distance function may be smaller than unity.

Caves *et al.* (1982) define an input-based Malmquist productivity index relative to a single technology Φ^t (in (16)) and Φ^{t+1} (in (17)), as follows:

$$M_i^t = \frac{D_i^t(X^{t+1}, Y^{t+1})}{D_i^t(X^t, Y^t)}; \tag{16}$$

$$M_i^{t+1} = \frac{D_i^{t+1}(X^{t+1}, Y^{t+1})}{D_i^{t+1}(X^t, Y^t)}. \tag{17}$$

The values of M_i^t and M_i^{t+1} may be smaller, equal or greater than one, depending on whether productivity growth, stagnation or productivity decline has occurred between periods t and $t+1$. In general, M_i^t and M_i^{t+1} yield different productivity numbers since their reference technologies are different.

The Malmquist index was treated as a theoretical one until its enhancement by Färe et al. (1994). A major contribution of this paper was to relax the efficiency assumption and use DEA models for the calculation of the distance functions embodied in the Malmquist index.

Färe et al. (1994) defined an input-oriented productivity index as the geometric mean of the two Malmquist indexes referring to the technology at time periods t and $t+1$ (i.e., (16) and (17)), yielding the following Malmquist-type measure of productivity:

$$M_i^{t,t+1} = \left[\frac{D_i^t(X^{t+1}, Y^{t+1})}{D_i^t(X^t, Y^t)} \cdot \frac{D_i^{t+1}(X^{t+1}, Y^{t+1})}{D_i^{t+1}(X^t, Y^t)}\right]^{1/2} \tag{18}$$

Another major achievement of Färe et al. (1994) was to show how to decompose the index $M_i^{t,t+1}$ into an index of technical efficiency change and an index reflecting the change in the frontier of the production possibility set, i.e. an index of technical (or technological) change. These components are obtained by rewriting the index in (18) as follows:

$$M_i^{t,t+1} = \frac{D_i^{t+1}(X^{t+1}, Y^{t+1})}{D_i^t(X^t, Y^t)} \cdot \left[\frac{D_i^t(X^{t+1}, Y^{t+1})}{D_i^{t+1}(X^{t+1}, Y^{t+1})} \cdot \frac{D_i^t(X^t, Y^t)}{D_i^{t+1}(X^t, Y^t)}\right]^{1/2} \tag{19}$$

The ratio outside the bracket measures the input technical efficiency change between time periods *t* and *t+1*. The geometric mean of the two ratios inside the bracket captures the technological change (or shift in technology) between the two periods, evaluated at the input-output levels at *t*, i.e. (X^t, Yt), and at *t+1*, i.e. (X^{t+1}, Y^{t+1}). Overall, for the Malmquist index defined in (19), which is based on distance functions, improvements in productivity yield input Malmquist indexes with values smaller than unity.

To make the interpretation of the index easier, we will calculate the index in terms of efficiency measures, such that improvements in productivity yield an index greater than unity. As explained in Fare and Lovell (1978), the input distance function is the reciprocal to Farrell's (1957) input measure of technical efficiency, which can be obtained using model (7). For a DMU j_0 with an input and output vector (X,Y), this can be stated by the following equality:

$$\theta^C = \frac{1}{D_i(X,Y)}. \tag{20}$$

The results of the Malmquist index reported in section 4 correspond to the calculations based on efficiency measures, such that productivity improvements, as well as technical efficiency improvements and improvements in location of the frontier of the PPS correspond to indexes greater than one.

For a comprehensive review of the literature on the theoretical developments and applications on the Malmquist index see Fare *et al.* (1998).

3. DATA

A key feature of the analysis in the present paper is the use of panel data. To be specific, we use data for 13 Portuguese universities for the years 2003, 2004 and 2005. Cost data and information about research output refer to calendar years, while data on student numbers refer to the academic years 2003-04 through 2005-06.

Our costs variable is the total level of university expenditures, both current and capital, obtained from the *Conta Geral do Estado* published annually by the *Ministério das Finanças e da Administração Pública*. Total expenditure includes operational expenditure and capital expenditure. Total expenditures is not an ideal measure, because the spending of universities includes items such as catering and accommodation costs that can vary substantially from one institution to another for reasons not connected with the quantity of educational outputs. Data on these 'hotel' expenditures are not available, however, and so we assume that they vary in proportion with other costs.

Student numbers data are available from the website of the *Observatório da Ciência e do Ensino Superior* (http://www.oces.mctes.pt) which is part of the *Ministério da Ciência, Tecnologia e Ensino Superior*. These are published separately for the undergraduate, master and doctoral levels of study. We considered these three types of students separately since we were interested in analysing the expenditure associated with each type of educational level. As an alternative, one could have separated students according to the subject areas attended (e.g. sciences, social sciences, arts and humanities). This avenue was followed by Warning (2004) that separated graduates and publications into two groups (sciences and social sciences).

In many previous studies, research funding has been used as a measure of research activity. This has the advantage of providing a quality-adjusted measure of the level of research output, but it also carries a disadvantage in that research funding should properly be regarded as an input rather than an output. In the case of Portugal, a measure of research funding is not available. We therefore use a straightforwardly available variable, namely the number of publications achieved by each university in the calendar year. Data on this may be obtained from the Institute for Scientific Information (ISI) by using the advanced search facility on the Web of Knowledge website. Some caveats must attach to this measure. In particular the coverage of the ISI data is incomplete, and the measure that is obtained is not adjusted for the quality of research output. Nevertheless, this variable likely captures the broad variation in research intensity that is to be observed in the Portuguese university system. It is expected that research in some areas can result in technology transfer between the university and its environment. We do not include technology transfer variables in the model because of data unavailability.

Descriptive statistics for the variables used in our analysis appear in Table 1. The raw data used in the analysis appear in the appendix. The standard deviation of all variables is quite high relative to the mean, indicating a considerable amount of diversity within the university sector in Portugal. The degree to which institutions are comprehensive in their provision of teaching and research clearly varies, with some institutions concentrating much of their activity on undergraduate level teaching while others are substantially engaged in research and teaching at all levels. In terms of costs, the largest institution (Porto) is some 14 times bigger than the smallest (Madeira). The heterogeneity of institutions is certainly sufficient to motivate an approach to the estimation of cost functions that is either non-parametric (like DEA) or accommodating of inter-institutional variation in parameters (like the random parameters stochastic frontier method).

TABLE 1

Descriptive statistics

Variable	Mean	Standard deviation	Minimum	Maximum
Costs (€'000)	81789	51649	12762	175839
Bachelors	10646	6720	2377	22542
Masters	681.9	592.5	0	1836
Doctoral	520.7	480.7	9	1666
Research	434.2	361.1	27	1234

4. RESULTS

4.1 Stochastic frontier analysis

In discussing the results of our empirical analyses, it should first be emphasised that the degree of collinearity between the output variables in the universities of Portugal is high. Our analysis includes four outputs: undergraduate degree students, masters degree students, doctoral degree students, and the number of research publications. The correlation coefficient between any two of these variables exceeds 0.9. This means that, while the results reported in this section provide a useful illustration of how the methods of frontier analysis can be employed in practice, they should be treated with a measure of caution.

The first set of results that we report here refer not to frontier estimates, but to a random parameters regression in which the half-normal residual is constrained to equal zero – that is, we assume no inefficiency. The results of this specification are reported in Table 2. Costs (measured in €'000) are modelled as a linear function of our four outputs. The estimates imply that there are considerable differences between the costs of tuition for bachelor level students (€2000 per student per annum in an average institution) and masters students (around €30000). Costs attached to doctoral study are relatively low, at around €10000 per annum. The cost attached to the production of research publications is estimated at about €25000. We are somewhat sceptical about the high figure estimated for masters students, and note that, in view of the multicollinearity present in the data, some prudence is required in interpreting these results[2].

The nature of a random parameters exercise is that the estimates provided above refer to an average institution, but these costs may differ – sometimes markedly – across institutions. Table 3 shows how the parameters of the model differ across the universities in the sample.

[2] The log likelihood associated with the regression is -404.63. We do not report log likelihoods elsewhere in the paper because different convergence criteria have been used for the various models, this making a comparison of the log likelihoods unhelpful.

TABLE 2

Estimates of the random parameters model

Variable	Coefficient	z statistic
Means for random parameters:		
Constant	13957.106	6.05
Bachelors	2.363	4.58
Masters	37.619	5.69
Doctoral	11.197	1.31
Research	26.866	2.70
Scale parameters for the distributions of random parameters:		
Constant	4877.407	3.85
Bachelors	0.029	0.31
Masters	2.900	1.91
Doctoral	0.760	0.45
Research	1.036	0.46
σ	6462.593	9.88

TABLE 3

Institution-specific coefficients, random parameters model

University	Constant	Bachelors	Masters	Doctoral	Research
Açores	12645	2.362	37.856	11.204	26.855
Algarve	19863	2.363	37.863	11.190	27.048
Aveiro	11484	2.359	36.723	11.170	26.667
Beira Interior	14161	2.366	37.848	11.007	26.667
Coimbra	8874	2.357	37.687	10.918	26.723
Évora	10455	2.357	36.194	11.198	27.034
Lisboa	15036	2.362	38.332	11.113	26.846
Madeira	8080	2.366	37.360	11.373	26.966
Minho	18721	2.367	38.445	11.366	26.793
Nova Lisboa	17255	2.369	38.469	11.600	27.090
Porto	9044	2.372	34.785	11.112	26.975
Técnica Lisboa	15160	2.368	38.872	11.193	26.877
Trás-os-Montes e Alto Douro	16043	2.371	37.104	10.997	27.021

It is easily seen that, while the constant varies quite substantially, the costs attached to the various outputs being considered are remarkably stable across institutions. In this respect, our findings here differ from those of earlier studies conducted in other countries. The fixed cost component represented by the constant, does however vary across universities, being unusually low in Madeira, Coimbra and Porto – these being respectively the smallest and the two largest universities. This finding suggests that the linear approach adopted here does not fully capture scale effects. Note that the unusually high cost value for master students is not easily explained within the Portuguese context.

We next estimate a frontier model with a random parameters on the constant. The coefficients of this model are reported in Table 4.

TABLE 4

Estimates of the stochastic frontier random parameters model

Variable	Coefficient	z statistic
Means for random parameters:		
Constant	19642.150	19.16
Bachelors	1.977	9.38
Masters	33.690	13.25
Doctoral	5.175	1.43
Research	19.753	5.62
Scale parameters for the distributions of random parameters:		
Constant	0.000	0.00
Bachelors	0.002	0.08
Masters	0.000	0.00
Doctoral	0.000	0.00
Research	0.000	0.00
σ	10631.735	7.21
λ	3.129	5.26

It is readily seen from this table that the coefficient estimates remain broadly similar to those obtained earlier. The scale parameters for the distribution of random effects are now all insignificant, however, and indeed there is no variation across institutions in the random parameters (including that on the constant term). All of this inter-institutional variation is now accounted for by differences in the one-sided residual term that is inefficiency. The efficiencies are reported in Table 5.

TABLE 5

Efficiency estimates, random effects stochastic frontier model

University	2003	2004	2005
Açores	0.894	0.991	0.763
Algarve	0.923	0.889	0.824
Aveiro	0.825	0.916	0.820
Beira Interior	0.745	0.973	0.783
Coimbra	0.752	0.954	0.954
Évora	0.750	0.921	0.878
Lisboa	0.901	0.777	0.887
Madeira	0.931	0.830	0.822
Minho	0.959	0.882	0.855
Nova Lisboa	0.932	0.944	0.848
Porto	0.899	0.910	0.747
Técnica Lisboa	0.749	0.811	0.864
Trás-os-Montes e Alto Douro	0.822	0.857	0.897

Overall the efficiency scores are high, and there seem to be few institutions with scores that are consistently below the norm. Only Técnica Lisboa and Trás-os-Montes fail to achieve a score of 0.9 or higher in any year, and only Beira Interior has more than one year in which the score is less than 0.8. One might speculate on the reasons for this. In 2003 Trás-os-Montes only had 63% of the places available for undergraduate students filled[3]. Although this number increased to around 80% in subsequent years, this may be one of the causes for the low efficiency detected for this university. The case of Beira Interior is identical, with only 63% of the places available for undergraduate students filled in 2005, although this number had been higher in the previous years (around 80% in 2003 and 2004).

In relation to Técnica Lisboa, the percentage of places for undergraduate students filled with applicants is above 90% in all years, so the sources of inefficiency are different. The lowest efficiency figure for this university corresponds to the year 2003. This result is in line with the DEA analysis, and the main cause of this inefficiency seems to be associated with the reduced number of masters students (which can be verified with a comparison to Porto, that has similar cost but dominates Técnica Lisboa in all other dimensions, as can be seen in the Appendix. The ratio of staff to students in the largest faculty of Universidade Técnica Lisboa (Instituto Superior Técnico) is quite high, which may also explain the inefficiency detected in the analysis.

Some limited discussion of scale and scope effects is possible using the results obtained above, though it should be borne in mind that the functional form that we have employed imposes some restrictions. In the discussion that follows, we use the definitions of scale and scope concepts employed by Baumol *et al.* (1982) and Johnes and Johnes (2006). For an institution with mean levels of all outputs, ray economies of scale are measured as 1.355, and global economies of scope are evaluated at 0.796. These figures suggest that both scale and scope economies remain unexhausted, a finding that clearly applies *a fortiori* to the case of the smaller institutions in the sample. (Note that product-specific returns to scale are constrained to unity when the cost function is linear.)

To summarise the findings of this section, the institution-specific parameters of a linear cost function have been estimated using panel data and allowing for inter-institution variation in technical efficiency. Costs associated with student numbers are, we estimate, highest for masters students. Fixed costs vary across institutions, though it would appear that this is for reasons of efficiency rather than any irreducible factor. That said, the general level of technical efficiency appears to be high in Portuguese institutions. The reader should be circumspect in

[3] In Portugal the government sets annually the maximum number of places available in each course for public universities. Universities compete to attract the best students and fill all the places available.

interpreting all of these results, given the small sample size and the considerable degree of multicollinearity in the data. Nevertheless the work reported here provides a powerful illustration of the managerial information that is offered by the random parameters stochastic frontier method.

In the next section, we employ DEA methods to evaluate technical efficiency. The method of Malmquist indices will then be used to measure changes in efficiency over the time period covered by our data. This will allow the results obtained above to be compared with those that emerge from a traditional non-parametric approach.

4.2 Data Envelopment Analysis

Table 6 reports the DEA results, with an input orientation, considering one input (total expenditure) and four outputs (undergraduates, Master and Doctoral students, and research output measured by the number of publications indexed in the ISI database). The efficiency measures reported are TE (evaluated with a CRS model) and its components of PTE (evaluated with VRS) and scale efficiency.

TABLE 6

DEA efficiency estimates

	2003			2004			2005		
University	CRS	VRS	Scale	CRS	VRS	Scale	CRS	VRS	Scale
Açores	0.72	0.87	0.83	0.68	0.71	0.97	0.57	0.69	0.82
Algarve	0.59	0.78	0.76	0.65	0.70	0.93	0.62	0.76	0.81
Aveiro	1	1	1	1	1	1	1	1	1
Beira Interior	0.97	0.97	1	0.72	0.81	0.89	0.79	0.84	0.93
Coimbra	0.999	1	1	1	1	1	1	1	1
Évora	1	1	1	1	1	1	1	1	1
Lisboa	1	1	1	0.94	0.95	0.99	1	1	1
Madeira	1	1	1	1	1	1	1	1	1
Minho	1	1	1	0.77	0.77	0.99	0.92	0.94	0.98
Nova Lisboa	0.85	0.85	1	0.91	0.94	0.97	0.86	0.86	1
Porto	1	1	1	1	1	1	1	1	1
Técnica Lisboa	0.88	0.89	0.99	1	1	1	1	1	1
Trás-os-Montes e Alto Douro	0.83	0.86	0.97	0.92	1	0.92	0.95	0.96	0.99

Aveiro, Évora, Madeira, Porto and Coimbra are efficient in the three years considered. Açores, Algarve, Beira Interior and Trás-os-Montes are inefficient in all years, and it was found evidence of the existence of scale inefficiency (<95%) in these universities at least in one of the years analysed. These universities tend to have the lowest percentages of places available for undergraduate students filled by applicants of higher education courses. Açores and Algarve were in the last positions of the rank of places filled in the three years analysed, whereas Trás-os-montes performed badly in this dimension in 2003 and Beira Interior performed badly in 2004 and 2005.

Nova de Lisboa was also inefficient in all years, but with no evidence of scale inefficiencies. The other universities have a mixed profile: Técnica de Lisboa was inefficient in 2003, Lisboa was inefficient in 2004, but efficient in the other two years. Minho efficient in 2003, inefficient in 2004, but has managed to improve efficiency in 2005, although it did not achieve yet fully efficiency.

In relation to the cost estimates associated with the production of outputs, the values obtained for each institution using model (11) are shown on Table 7. The input considered was the average cost in years 2003, 2004 and 2005, and the outputs were the average number of undergraduate, master and doctoral students and the average number of publications in the same period. Particular care is needed in interpreting the results, since the sample size is small (consisting of only 13 universities) and the degree of correlation between the output variables is high. The analysis reported below should be seen as exploratory and illustrative of the potential of the DEA technique to provide information that can uncover issues of interest to managers and policy makers.

TABLE 7

Institution-specific cost per unit of output.

University	Constant	Undergraduates	Masters	Doctoral	Research
Açores	1,765,064 €	3,945 €	20,000 €	2,000 €	20,000 €
Algarve	1,765,064 €	3,945 €	20,000 €	2,000 €	20,000 €
Aveiro	3,065,626 €	3,160 €	20,000 €	20,000 €	20,000 €
Beira Interior	0 €	5,262 €	2,000 €	10,935 €	2,000 €
Coimbra	0 €	3,326 €	20,000 €	20,000 €	20,000 €
Évora	1,846,085 €	3,919 €	20,000 €	20,000 €	2,000 €
Lisboa	3,065,626 €	3,160 €	20,000 €	20,000 €	20,000 €
Madeira	3,065,626 €	3,160 €	20,000 €	20,000 €	20,000 €
Minho	0 €	5,139 €	11,756 €	2,000 €	2,000 €
Nova Lisboa	3,065,626 €	3,160 €	20,000 €	20,000 €	20,000 €
Porto	0 €	3,326 €	20,000 €	20,000 €	20,000 €
Técnica Lisboa	0 €	3,326 €	20,000 €	20,000 €	20,000 €
Trás-os-Montes e Alto Douro	654,955 €	4,867 €	2,000 €	20,000 €	2,000 €

The cost analysis based on the use of DEA reveals three different clusters regarding cost behaviour. The first cluster corresponds to the largest universities in Portugal: Porto, Técnica Lisboa and Coimbra. The criteria used to determine the size of the universities was the number of students in the period 2003-2005. These universities had, on average, more that 20.000 students registered in this period. The costs associated with master and Doctoral students and with publications are quite high (in fact, the cost estimates are equal to the upper bound of the weights imposed in the DEA model). This reveals the focus of these universities on postgraduate students and research. The fixed costs of these universities are negligible.

The second cluster identified corresponds to the Universities of Lisboa, Nova Lisboa, Aveiro and Madeira. With the exception of Madeira, these correspond to large universities (4th, 6th and 7th positions of the rank based on the criteria described above, whereas Madeira is in the last position of the rank). These universities also value highly postgraduate students and research, but in contrast with the largest universities, these have high fixed costs.

The third cluster corresponds to Açores and Algarve that are among the smallest universities (11th and 12th positions of the rank). These universities have high fixed costs and value highly master students and research but do not focus on doctoral students.

The remaining universities cannot be easily assigned to any of the clusters above, as there cost estimates are unique. However, the profile of Évora and Trás-os-montes is to some extent similar to the third cluster: they are small universities (8th and 9th position of the rank), with high fixed costs. Évora favours teaching masters and doctoral students to the detriment of publishing articles, whereas Trás-os-Montes only differs from Évora as it favours undergraduate students instead of master students. Beira Interior and Minho could be said to form a separate cluster. These universities have negligible fixed costs and their mission seems to be related to undergraduate teaching. In terms of post-graduate teaching, Beira Interior focuses on doctoral students, whereas Minho favours masters.

4.3 Malmquist index

The results of the Malmquist index calculated using 2003/04 as the base period are reported on Table 8.

Globally, the results of the Malmquist index indicate that the productivity of the Portuguese universities increased in 2004/05, mainly due to the technological improvement observed in the frontier. However, this improvement path was not maintained in 2005/06, as the productivity level declined to values lower than those of the base period (2003/04). Between 2003 and 2006, the average efficiency of the Portuguese universities remained unchanged.

In relation to the productivity change of each university, we will separate the analysis in three groups, corresponding to the Universities considered efficient in the three years analysed (Aveiro, Coimbra, Évora, Madeira e Porto), the universities inefficient in the three years analysed (Açores, Algarve, Beira Interior, Nova de Lisboa, Trás-os-Montes), and those with a mixed profile (Lisboa, Minho, Técnica

TABLE 8

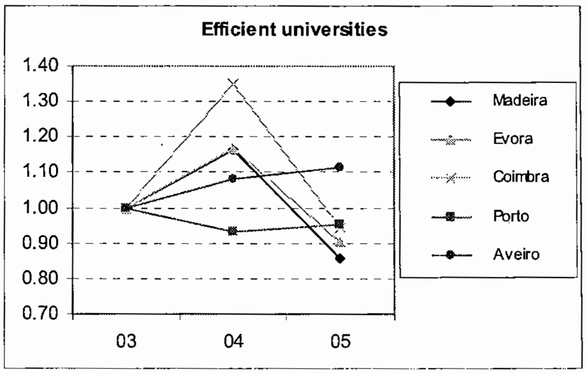
Malmquist index and sub indexes for efficiency change and technological change.

University	2003/04 →2004/05			2003/04 →2005/06		
	MI	Eff.C	Tec.C	MI	Eff.C	Tec.C
Açores	**1.11**	0.94	1.18	**0.70**	0.79	0.89
Algarve	**1.02**	1.10	0.93	**1.01**	1.04	0.97
Aveiro	**1.09**	1	1.09	**1.11**	1	1.11
Beira Interior	**0.94**	0.75	1.27	**0.71**	0.81	0.88
Coimbra	**1.35**	1	1.35	**0.94**	1	0.94
Évora	**1.16**	1	1.16	**0.90**	1	0.90
Lisboa	**0.85**	0.94	0.90	**1.01**	1	1.01
Madeira	**1.16**	1	1.16	**0.85**	1	0.85
Minho	**0.97**	0.77	1.27	**0.83**	0.92	0.89
Nova Lisboa	**1.03**	1.08	0.95	**0.98**	1.01	0.97
Porto	**0.93**	1	0.93	**0.95**	1	0.95
Técnica Lisboa	**1.06**	1.14	0.93	**1.02**	1.14	0.89
Trás-os-Montes e Alto Douro	**1.23**	1.10	1.12	**1.02**	1.15	0.89
AVERAGE	**1.07**	**0.99**	**1.10**	**0.93**	**0.99**	**0.94**

de Lisboa). For each university, its productivity level observed in the base period (year 2003/04) is attributed a value of 1. The productivity change is illustrated in Figures 1, 2 and 3.

FIGURE 1

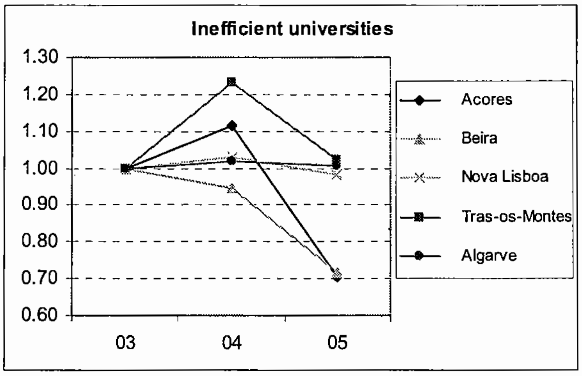
Illustrative representation of the Malmquist index for efficient universities in the three years analysed



In relation to the efficient universities, we can observe that only one university increased productivity between 2003/04 and 2005/06 (Aveiro). Two universities remained rather stable in productivity terms (Porto and Coimbra), whereas the productivity of Madeira and Évora decreased. However, all these universities (expect Porto) improved in 2004/05, but not all managed to sustain this improvement in subsequent years.

FIGURE 2

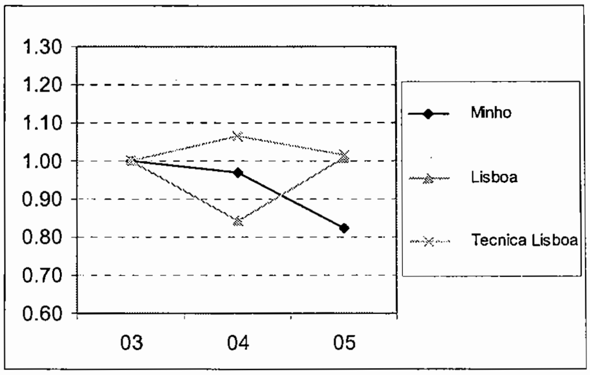
Illustrative representation of the Malmquist index for inefficient universities in the three years analysed



The analysis of the inefficient universities revealed that the most problematic cases are Açores and Beira Interior, both showing a significant productivity decrease between 2003/04 and 2005/06. Nova de Lisboa and Algarve kept their productivity levels between 2003/04 and 2005/06, and trás-os-montes increased productivity in 2004/05, but these gains were lost in 2005/06, such that productivity levels in 2005/06 were similar to those of 2003/04.

FIGURE 3

Illustrative representation of the Malmquist index for universities that were classified both as efficient and inefficient in the years considered.



Finally, for the three universities with an undefined profile in terms of efficiency in the three years analysed, we can conclude that Minho has lost productivity between 2003/04 and 2005/06, but Lisboa and Técnica de Lisboa have productivity levels in 2005/06 similar to those of 2003/04 (although Lisbon had a particularly bad year in productivity terms in 2004/05, but it has recovered the productivity lost in the following year).

5. CONCLUSIONS

In this paper we present some results from an assessment of Portuguese universities using a parametric stochastic frontier method and a non-parametric DEA method. We assessed 13 public universities in total, which represent the public universities in the country. The assessment involved the estimation of efficiency for the three time periods being considered (2003, 2004 and 2005) and also an estimate of how each factor being considered explained total costs of universities (the factors that we considered were number of students of three types – undergraduates, master and doctoral students – and number of ISI publications as a variable reflecting the research intensity at the university). These two outputs of the analyses (efficiency scores and cost estimates) were produced both for the non-parametric and parametric approaches. Aside from that in the non-parametric DEA analysis we computed Malmquist indices and decomposed them into efficiency change and technological change.

Conclusions from both methods point largely to the same direction especially as far as worst performers are concerned. The smaller congruence between methods for best performers may be a problem inherent to the DEA method that, by giving flexibility to universities in assigning weights, leads some universities to achieve efficiency by neglecting some outputs in the assessment. This was a problem not completely explored in the present paper that may be the subject of further research. Nevertheless in the DEA analysis the introduction of weight restrictions was imposed as a way to analyse the cost structure of universities, i.e., the contribution of each output in explaining total expenses of universities. The analysis performed tried to parallel what is done in stochastic frontier analysis but the results in this case are not coincident. This is particularly true as far as actual unitary costs are concerned, but not true in terms of magnitude of values. Both methods agreed that in general teaching undergraduates is less costly than teaching master students, and that research is about as costly as teaching a master student. More divergence appeared regarding the magnitude of values for doctoral students.

Malmquist indices revealed that universities in Portugal increased their productivity from the academic year 03/04 to the academic year 04/05, and this was mainly explained by technological progress. In contrast, from 03/04 to 05/06 there was productivity decline also mainly explained by technological regress. More periods would be required for a thoughtful analysis of productivity change in Portuguese public universities. This will also be addressed in future research as data becomes available.

In summary we would like to stress the fact that this was mainly an illustrative exercise and more accurate data are required (especially as far as cost data is concerned) so that more reliable conclusions can be drawn from an analysis of the performance of Portuguese universities. Since the data collection process in education in general is currently in the political agenda of the Portuguese state, we hope to continue this study in the future with better data and a larger panel.

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References

Agasisti, T. and Johnes, G. (2006) Heterogeneity and the evaluation of efficiency: the case of Italian universities, mimeo, Lancaster University Management School.

Aigner, D.J., Lovell, C.A. and Schmidt, P. (1977) Formulation and estimation of stochastic frontier production function models, *Journal of Econometrics*, 6, 21-37.

Banker, R.D., Charnes, A. and Cooper, W.W. (1984) Some models for estimating technical and scale inefficiencies in data envelopment analysis, *Management Sciene,* 30, 1078-1092.

Baumol, W.J., Panzar, J.C. and Willig, R.D. (1982) *Contestable markets and the theory of industry structure*, San Diego: Harcourt Brace Jovanovich.

Caves, D.W., Christensen, L.R., and Diewert, W.E. (1982) The Economic Theory of Index Numbers and the Measurement of Input, Output and Productivity, *Econometrica,* 50, 1393-414.

Charnes, A., Cooper, W.W. and Rhodes, E. (1978) Measuring the efficiency of decision-making units, *European Journal of Operational Research*, 2, 429-444.

Cohn, E.R., Sherrie, L.W. and Santos, M.C. (1989) Institutions of higher education as multi-product firms: economies of scale and scope, *Review of Economics and Statistics*, 71, 284-290.

Färe, R. and Lovell, C.A.K (1978) Measuring the Technical Efficiency of Production, *Journal of Economic Theory* 19, 150-162.

Färe, R., Grosskopf, S., Lindgren, B., and Roos, P. (1994) Productivity Developments in Swedish Hospitals: A Malmquist Output Index Approach, in Charnes A., Cooper W. W., Lewin A. Y. , and Seiford L. M. (eds) *Data Envelopment Analysis: Theory, Methodology and Applications*, Boston: Kluwer Academic Publishers, pp.253-272..

Färe, R., Grosskopf, S., and Roos, P. (1998) Malmquist Productivity Indexes: a Survey of Theory and Practice, Pp. 127-90 in Färe R., Grosskopf S., and Russell, R.R. (eds) *Index Numbers: Essays in Honour of Sten Malmquist*, Kluwer Academic Publishers, pp.127-190.

Farrell, M.J. (1957) The measurement of productive efficiency, *Journal of the Royal Statistical Society Series A*, 120, 253-290.

Greene, W. (2005) Reconsidering heterogeneity in panel data estimators of the stochastic frontier model, *Journal of Econometrics*, 126, 269-303.

Izadi, H., Johnes, G., Oskrochi, R., and Crouchley, R. (2002) Stochastic frontier estimation of a CES cost function: the case of higher education in Britain, *Economics of Education Review*, 21, 63-71.

Johnes, G. (1996) Multiproduct cost functions and the funding of tuition in UK universities, Applied Economics Letters, 3, 557-561.

Johnes, G., Johnes, J., Thanassoulis, E., Lenton, P. and Emrouznejad, A. (2005) *An exploratory analysis of the cost structure of higher education in England*, London: Department for Education and Skills Research Report 641.

Johnes, G. and Johnes, J. (2006) Higher education institutions' costs and efficiency: taking the decomposition a further step, mimeo, Lancaster University Management School.

Johnes, G. and Salas-Velasco, M. (2006) The determinants of costs and efficiencies where producers are heterogeneous: the case of Spanish universities, mimeo, Lancaster University Management School.

Johnes, G. and Schwarzenberger, A. (2006) Differences in cost structure and the evaluation of efficiency: the case of German universities, mimeo, Lancaster University Management School.

Jondrow, J., Lovell, C.A.K., Materov, I.S. and Schmidt, P. (1982) On the estimation of technical inefficiency in the stochastic frontier production function model, *Journal of Econometrics*, 19, 233-238.

Malmquist, S. (1953) Index Numbers and Indifference Surfaces, *Trabajos De Estatistica*, 4, 209-42.

OECD (2006) Reviews of National Policies for Education: Tertiary Education in Portugal, Examiners' Report, EDU/EC(2006)25, Paris: OECD.

Tsionas, E.G. (2002) Stochastic frontier models with random coefficients, *Journal of Applied Econometrics*, 17, 127-147.

Warning, S. (2004), Performance differences in German higher education: empirical analysis of strategic groups, *Review of Industrial organization 24*, 393-408.

APPENDIX

TABLE 9

Inputs and outputs in 2003.

University	Cost	Undergraduates	Masters	Doctoral	Research
Açores	21,716,737 €	2812	87	32	59
Algarve	49,602,323 €	3921	349	46	191
Aveiro	64,842,536 €	8110	452	347	532
Beira Interior	29,080,597 €	5017	154	127	58
Coimbra	121,895,558 €	19403	933	763	687
Évora	41,860,975 €	7454	211	233	91
Lisboa	119,939,187 €	16358	1345	1175	669
Madeira	12,177,669 €	2407	0	46	30
Minho	83,209,305 €	14878	477	225	359
Nova Lisboa	105,170,835 €	13214	958	704	573
Porto	150,461,227 €	22229	1814	1395	962
Técnica Lisboa	153,877,039 €	19131	1630	921	855
Trás-os-Montes eAlto Douro	42,117,438 €	6612	94	35	80

TABLE 10

Inputs and outputs in 2004.

University	Cost	Undergraduates	Masters	Doctoral	Research
Açores	22,855,273 €	2641	176	31	42
Algarve	49,253,719 €	3430	356	26	191
Aveiro	69,670,003 €	7872	575	384	612
Beira Interior	30,227,286 €	5036	84	124	60
Coimbra	79,466,890 €	18267	667	657	693
Évora	44,220,790 €	7174	513	269	87
Lisboa	144,245,269 €	17459	1259	1178	730
Madeira	13,000,218 €	2441	139	9	27
Minho	86,726,505 €	14114	621	327	406
Nova Lisboa	106,220,569 €	12811	919	843	565
Porto	171,550,264 €	22542	1836	1489	1082
Técnica Lisboa	159,050,642 €	18760	1823	925	884
Trás-os-Montes e Alto Douro	41,648,783 €	6125	151	327	104

The cost values for Coimbra in 2004 do not include Faculdade de Ciências e Tecnologia.

TABLE 11

Inputs and outputs in 2005.

University	Cost	Undergraduates	Masters	Doctoral	Research
Açores	26,710,580 €	2377	90	53	51
Algarve	50,563,626 €	3317	311	163	242
Aveiro	71,385,376 €	7819	617	378	725
Beira Interior	40,021,784 €	5096	95	114	68
Coimbra	123,308,507 €	17623	1111	776	750
Évora	45,563,268 €	6765	394	277	110
Lisboa	128,411,407 €	17077	952	1399	793
Madeira	14,961,230 €	2456	48	47	47
Minho	101,816,797 €	13552	646	687	462
Nova Lisboa	115,327,051 €	12529	892	876	707
Porto	174,731,737 €	21839	1777	1666	1234
Técnica Lisboa	166,549,743 €	18493	1824	935	1001
Trás-os-Montes e Alto Douro	43,101,844 €	6044	213	298	115

Resumo:

Este artigo aplica duas metodologias para a medição da eficiência de Universidades Portuguesas. É utilizado um modelo paramétrico de fronteiras estocásticas e um modelo não-paramétrico de data envelopment analysis (DEA). Para cada um dos métodos, são apresentados resultados relativos aos níveis de eficiência das universidades Portuguesas entre 2003 e 2005. São também obtidas estimativas do custo unitário de ensino dos alunos em cada nível de ensino (licenciatura, mestrado e doutoramento), bem como dos custos associados à produção de artigos científicos. Os resultados obtidos por ambos os métodos são semelhantes, em certa medida, principalmente no que diz respeito à identificação das universidades com piores níveis de desempenho. Também se apresenta a evolução dos níveis de produtividade das universidades Portuguesas entre 2003 e 2005.

Palavras-chave: Fronteira estocástica, data envelopment analysis, índice de Malmquist, universidades