School of Economics and Management
TECHNICAL UNIVERSITY OF LISBON

DILUTION AND DIVIDEND EFFECTS ON THE PORTUGUESE EQUITY WARRANTS MARKET*

José Eduardo Correia
CEFAGE - UE, Management Department, Évora University, Portugal

João Duque
Advance Research Center, School of Economics and Management (ISEG) - Technical University of Lisbon, Portugal

Abstract

The aim of this study is to analyse the impact of dilution and dividends on the goodness of fit of warrant pricing valuation models, to the Portuguese warrants market. In order to avoid modelling bias over the research design, and to test dividend and dilution effects we decided to keep this empirical research under the Black-Scholes framework. Therefore, four pricing models were used: the original Black-Scholes model and three derivations. Using these four models we empirically estimate values for actual warrant prices, computing the mean percentage error for each (the difference between model prices and market prices). We found that the original Black-Scholes model when adjusted to account for dilution as well as for dividends works best in the Portuguese market. The analysis uses data collected from the Euronext - Lisbon, between 1998 and 2000.

Key Words: Warrants, implied volatility, Black-Scholes Model, dilution effect, Portuguese market.

JEL Classifications: G13, G14

* Authors are grateful to Julia Sawiki (Nanyang Technological University) and to Professor Dean A. Paxson (University of Manchester) for their opportune comments on an earlier version of the paper.

Correspondence Address: José Eduardo Correia – CEFAGE - UE, Management Department, Évora University, Largo dos Colegiais, 2, 7000 Évora – Portugal.E-mail: jec@uevora.pt

1. INTRODUCTION

Since the early sixties equity warrants have been under a constant interest of research in finance. With the seminal paper of Black-Scholes (1973) the warrants pricing literature had a new blow of a powerful methodology and got a strong framework to find whether other effects are actually observed. As long term options, warrants are expected to suffer the impact of dividends as well the so-called dilution effect.

Although dilution and dividends have been a constant concern of researchers, only in literature we find sophisticated methods to deal with these problems, namely in Merton (1973), Roll (1977), Galai and Schneller (1978), Geske (1979, 1981), Whaley (1981), Lauterbach and Schultz (1990) or Schulz and Trautmann (1989 and 1994). These authors show that dilution and dividends have some impact on market prices for warrants. But are there similar effects in illiquid markets? The bias introduced by thin trading is so strong that it may be plausible that the typical effects that we notice in other warrant markets, namely the dilution and dividend effects, may not be observed in illiquid markets. We thought that it could be interesting to check empirically whether dividends and dilution have some impact on warrants market prices, using a quite illiquid market as the Portuguese.

In order to avoid modelling bias over the research design and in order to test only, dividend and dilution effects, we decided to develop our research exclusively within the Black-Scholes framework. We used four warrants pricing models: the original Black-Scholes model and three of its derivations. Using these four models we empirically estimate values for actual warrant prices, computing the mean percentage error, as the difference between model prices and market prices. It is supposed that the most efficient model shows the smallest percentage error. The analysis uses data collected from the Euronext - Lisbon[1], between 1998 and 2000.

We concluded that there is a need for adjusting the original Black-Scholes model to dilution and to dividends. Concerning the dilution effect, we used the adjusted Black-Scholes formula proposed by Lauterbach and Schultz (1990) and discuss the possibility of the warrant market price to already include this effect, as supported by Crouhy and Galai (1991).

In order to test the need for dividend adjustments we used two models: the adjustment of the dividends in a discrete way and the adjustment to the payment of dividends proposed by Merton (1973).

[1] Although the official name Portuguese stock exchange at the time when this research was carried out was BVLP - Bolsa de Valores de Lisboa e Porto, after the recent merger it became part of the Euronext group, and adopted the formal name of Euronext - Lisbon.

The paper is organized as follows: in section 2 we review the literature, next we present the methodology and the data used and finally we present the empirical results and the conclusions.

2. LITERATURE REVIEW

Until 1973, the literature on contingent stocks was substantially devoted to warrants. Over-the-counter (OTC) options didn't receive a lot of attention either by academics or research departments of financial institutions. This trend changed with the start of traded options in the floor of some American stock exchanges and with the publication of the seminal paper of Black and Scholes (1973). However, in their article the empirical validation of the equation was based on warrants. The shift of attention from warrants to options is partially explained by the additional difficulties that the study of warrants incorporates such as dilution and dividends.

Early warrant studies, such as Sprenkle (1961), Samuelson (1965), Chen (1970) and Bierman (1973), ignored the dilution effect and considered warrants equivalent to call options. After Black and Scholes (1973) there have been many empirical studies on option pricing valuation, but very few empirical studies on warrant markets.

Ferri, Kremer and Oberhelman (1986) studied the goodness of fit of the pricing models, using a sample of 50 warrants traded in the U.S.A. for nine days between 1983 and 1984. Lauterbach and Schultz (1990) compared the Black-Scholes model with the Constant Elasticity of Variance model (CEV), observing the daily price of 39 warrants between 1979 and 1980 in the U.S.A. Schulz and Trautmann (1989) studied 49 German warrants from 1979 to 1986, using weekly data. Stucki and Wasserfallen (1991) used five pricing options models to study equity warrants, through a sample of 2.100 weekly observations for 44 Swiss warrants from January 1986 to February 1987.

In the empirical studies on warrant pricing several alternative models have been used. Veld (2003) refers that these models try to overcome some of the assumptions used by Black-Scholes, such as:

- No dividend payments - Merton (1973) adjusted the Black-Scholes equation allowing the model to be used in dividend paying firms;
- The early exercise before maturity - Merton (1973) showed that the rational investors must only use the possibility of early exercise for call options just before the ex-dividend date. Other models such as Black (1975), Roll (1977), Geske (1979, 1981) and Whaley (1981), and the American Constant Variance diffusion model (Schulz and Trautmann, 1994) try to overcome both problems (early exercise and dividend effect);

– Constant volatility – in the Constant Elasticity of Variance (CEV) model the assumption of constant volatility is replaced by the constant elasticity of volatility assumption. In this model it is assumed, that the elasticity factor is defined in such a way that the volatility decreases as the price of the underlying stock increases.

Sidenius (1996) refers some examples of the difficulties of warrant valuation. One example is the constant volatility assumption, which is not very realistic for a warrant with duration of several years. The volatility of a warrant with long maturity is likely to change and therefore, the valuation model should be adjusted to take into account these changes on volatility.

In the pioneering studies on warrant pricing, it was assumed that the firm was completely financed by shares and warrants. As a consequence, Galai and Scheneller (1978) and Crouhy and Galai (1991) priced warrants and shares based on the total value of the firm. Leonard and Solt (1990), Lauterbach and Schultz (1990) and Schulz and Trautman (1994) tested the Black-Scholes model in pricing warrants while Noreen and Wolfson (1981), Ferri, Kremer and Oberhelman (1986), Sisson (1987), Lauterbach and Schultz (1990) and Hauser and Lauterbach (1996 and 1997) compared the results obtained by the Black-Scholes model with the results of other pricing models. They concluded that the Black-Scholes model is the most representative and it is more precise than other models (including the Constant Elasticity of Variance). In spite of similar results, the studies differ in their treatment of dividends and dilution effects.

Emanuel (1983) and Constantinides (1984) derived a valuation model and an optimal exercise strategy for the American warrants with payment of dividends. Cox and Rubinstein (1985) and Spatt and Sterbenz (1988) considered the hypothesis of a potential expansion of the firm, deriving the optimal exercise strategy for American warrants.

Kim and Young (1980) studied the efficiency of the warrants market observing the relationship between warrants and shares. The study involved a hedging strategy, which was based on a long position on the underlying stock and a short position on warrants. This strategy tends to minimize profits below a target rate instead of maximizing them. They developed a model for determining the optimal hedge ratio, which was based on the probability of the price of the share having a certain value in the warrant's expiration date. This probability is defined by a function, which considers the current prices of the share and the warrant and the exercise price. The empirical study considered 18 warrants traded between 1962 and 1977. They concluded that the profit of a hedging strategy with warrants is superior to the profit with a buy and hold strategy.

Wei (1995) evaluated the Nikkei 225 put warrants. He studied warrants traded in the Toronto Stock Exchange (Canada) and used several pricing models proposed by Dravid, Richardson and Sun (1993), Reiner (1991) and Wei (1992). Wei concluded that these models tend to undervalue the warrants relatively to their market prices. This under valuation tends to be stronger when warrants are deep in-the-money, the volatility of the underlying Index is high and the trading volume is also high.

Huang and Chen (2002) applied the stochastic volatility option pricing model of Hull and White (HW) to the covered warrants traded on the Taiwan Stock Exchange (TSE). They concluded that the HW model with implied volatility outperforms others in predicting the warrant prices, indicating that the pricing model incorporated with stochastic volatility feature can improve the pricing of warrants. Ukhov (2004) developed an algorithm for pricing warrants using stock prices, an observable variable and stock return variance. The algorithm is based on the variables used in the Black-Scholes option pricing formula, the number of shares outstanding, the number of warrants issued, and the number of shares of stock that each warrant entitles the owner to receive when exercised.

Lim and Terry (2003) created a formula do evaluate multiple series of warrants and compared the theoretical warrant price from their model with existing models, like Black-Scholes (1973) and Galai-Schneller (1978). They found a subtle slippage effect and also a cross dilution effect that caused the existing models, to be inappropriate for pricing such classes of multiple warrants.

Horst and Veld (2003) used the Black-Scholes, the Square Root model version of the CEV, and the Binomial model to price the call warrants based on long-term call options and found that call warrants are overvalued between 25 and 30 percent for all three models. The authors considered "that the overvaluation can be attributed to a behavioral preference of private investors for call warrants"

Loudon and Nguyen (2006) concluded that are a large excess warrant premium and provided evidence that it is significantly related to the identity of the warrant issuer, even after taking into account important liquidity and hedging factors.

3. METHODOLOGY

In this study we are particularly interested on how strong are the empirical impacts of dividends and dilution effect on pricing warrants. In order to keep the research design out of any other effect we decided to use a very simple model as the Black and Scholes (1973). We could then obtain a minimum of implied parameters from the basic model, and introduce the referred effects: dividends and dilution. Of course the same approach could be developed using a different model, but as the Black-Scholes model keeps the number of unobservable parameters to a minimum, and because it is a benchmark on pricing in similar studies, we also decided to use it as a reference model.

The methodology of this study consists of obtaining theoretical values for the four pricing models selected (Black-Scholes and three of its derivations) and to compute a mean percentage error for each one of them relative to the observed market prices[2]. We assume that the best model to price equity warrants is the one that presents the smallest percentage error. This criterion is a common procedure to previous warrant pricing studies.

To test the need for modelling adjustments when assuming dividend paying firms, two derivations of the Black-Scholes model were used:

1. Adjustment to dividends in a discrete way, that is, the adjustment for dividends is made by replacing the price of the underlying stock, S, for the price of the stock minus the net present value of the dividends that will be paid until maturity, Sd, as it was done by Lauterbach and Schultz (1990);
2. Adjustment according to Merton (1973) model where the dividends are supposed to be paid continuously until maturity according to a constant dividend yield.

In terms of the dilution effect, a derivation of the Black-Scholes model proposed by Lauterbach and Schultz (1990) was applied.

Stucki and Wasserfallen (1991) applied the arbitrage conditions, in order to prevent arbitrage opportunities in the database. Warrant prices should satisfy the same arbitrage conditions that are applied to call option prices, and pricing models are only meaningful when those conditions are not violated. There exist at least three conditions for the minimal value of a warrant that should be tested:

1 – The value of the warrant (W) should be at least equal to the maximum between zero and the difference between the current underlying stock price (S) and the exercise price (X):

$$W \geq \max(S - X, 0) \tag{3.1}$$

This equation defines lower bound for an American contract.

2 – The value of the warrant should be equal or greater than the maximum between zero and the difference between the current price of the underlying stock and the present value of the exercise price:

$$W \geq \max\left(S - Xe^{-r(T-t)}, 0\right) \tag{3.2}$$

[2] Since the number of warrants considered in the study is very small (only six warrants), we study the entire set of warrants as well as each one of them.

where T is the exercise date and r is the risk-free interest rate. This equation defines lower bound for an European contract.

3 – Taking into account the effects of dividends on the price of the underlying stock one can impose stricter limits on the previous condition. Replacing the price of the underlying stock, S, for the price of the stock minus the net present value of the dividends that will be paid until maturity, Sd:, we will get:

$$W \geq \max\left(S_d - Xe^{-r(T-t)}, 0\right) \tag{3.3}$$

In order to select warrant prices for this study we tested whether these three arbitrage conditions were violated. Whenever we detected some violation, the corresponding observation was excluded from the sample.

As explained previously, we used four warrant pricing models in order to test the goodness of fit for the Portuguese warrants market:

1. The Black-Scholes model – BS:

$$W = \left[SN(d_1) - Xe^{-r(T-t)}N(d_2)\right] \times \gamma \tag{3.4}$$

$$d_1 = \frac{\ln\frac{S}{X} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}} \tag{3.5}$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)} \tag{3.6}$$

where:

W = value of the warrant;

S = stock price;

X = exercise price;

γ = exercise ratio = number of shares of the underlying stock that can be bought / sold with each warrant;

r = risk-free interest rate;

T = expiration date of the warrant;

$(T-t)$ = time-to-maturity of the warrant;

σ = stock return volatility;

N(.) = cumulative standard normal distribution function.

2. *Black-Scholes model adjusted for dividends in the discrete form (adjusting the underlying stock price with the net present value of the dividends) – Bsdiv:*

$$W = \left[S_d N\left(d_1^{div}\right) - Xe^{-r(T-t)} N\left(d_2^{div}\right) \right] \times \gamma \tag{3.7}$$

$$d_1^{div} = \frac{\ln \frac{S_d}{X} + \left(r + \frac{\sigma^2}{2} \right)(T-t)}{\sigma\sqrt{T-t}} \tag{3.8}$$

$$d_2^{div} = d_1^{div} - \sigma\sqrt{T-t} \tag{3.9}$$

where:

$$S_d = S - \sum_{i=1}^{n} D_i e^{-r(T_i - t)} \tag{3.10}$$

D_i = the ith dividend;
T_i = time moment when the ith dividend is paid;

3. *Black-Scholes model adjusted for dividends in the continuous form, proposed by Merton (1973) – BS-M;*

$$W = \left[Se^{-d(T-t)} N\left(d_1^{M}\right) - Xe^{-r(T-t)} N\left(d_2^{M}\right) \right] \times \gamma \tag{3.11}$$

$$d_1^{M} = \frac{\ln \frac{S}{X} + \left(r - d + \frac{\sigma^2}{2} \right)(T-t)}{\sigma \sqrt{T-t}} \tag{3.12}$$

$$d_2^{M} = d_1^{M} - \sigma\sqrt{T-t} \tag{3.13}$$

where:

d = dividend yield. (3.14)

4. *The Lauterbach and Schultz (1990) - Black-Scholes Dilution-Adjusted model – BSAD.*

Some authors multiply the Black-Scholes formula by the dilution factor (N/((N/γ)+M)), where M is the number of warrants issued and N is the number of shares outstanding. However, such a procedure is naive and assumes that market prices for both markets (the warrants markets as well the underlying security market) are absolutely segregated, which is not the case. Some other modifications are needed. Lauterbach and Schultz (1990) suggested replacing the underlying stock price, S, by the equity value per share of common stock, S_v; replacing the volatility, σ, by the equity volatility, σ_v; and finally multiplying the result by the dilution factor. According to Lauterbach and Schultz (1990) the value of the equity warrant is then given by:

$$W = \frac{N}{N/\gamma + M}\left[\left(S_d + \frac{M}{N}W\right)N(d_1^{AD}) - Xe^{-r(T-t)}N(d_2^{AD})\right] \tag{3.15}$$

$$d_1^{AD} = \frac{\ln\left[\frac{\left(S_d + \frac{M}{N}W\right)}{X}\right] + \left(r + \frac{\sigma_v^2}{2}\right)(T-t)}{\sigma_v\sqrt{(T-t)}} \tag{3.16}$$

$$d_2^{AD} = d_1^{AD} - \sigma_v\sqrt{(T-t)} \tag{3.17}$$

where:

$\left(S_d + \frac{M}{N}W\right) = S_v$ = equity value that the stock price is adjusted to the dividends;

σ_v = the equity volatility that equals the volatility of the total asset instead of the volatility of the underlying share price.

It is clear that for firms that do not pay dividends, the warrant theoretical prices were equivalent in the first three models, since dividends is the only variable that differs.

In order to estimate the volatility, we started by computing the implied volatility from each warrant market price. As in other studies, we assumed that the Black-Scholes model holds and both, the stock and options markets, are efficient.

Then we used the implied volatility as an appropriated estimator for future market volatility. The implied volatility was calculated using the Newton-Raphson iterative process. The iterative process stopped whenever the simulated warrant price differed less than 0.001% from the warrant market price. We did not remove observations with very low market prices, very deep in-the-money or out-of-the-money or close to the maturity, because in Duque (1994) they were found to have significant information.

A similar process was developed using the four models, which lead us to compute four different implied volatilities per trading day.

Then we averaged the past five days implied volatility observations in order to feed the model when forecasting the future warrant price. This procedure was done for each of the models under study. Whenever the arbitrage conditions were not satisfied, implied volatilities could not be computed because the algorithm does not converge to a stable figure.

$$\sigma_t^k = \frac{\sum_{i=1}^{5} \sigma_{imp_{t-i}}^k}{5} \tag{3.18}$$

As equation (3.18) documents, for each day *t* the volatility to be fed into the model *k* (σ_t^k) equals the average implied volatility observed in the previous 5 days (σ_{impt-i}^k), being these implied volatilities computed by using model *k* [3].

The performance of each model is measured by *percentage error*:

$$\text{Percentage Error} = \frac{\text{model value - market price}}{\text{market price}} \times 100\% \tag{3.19}$$

The prices of the models are calculated for each warrant and for each daily observation. The mean of the percentage error is useful to determine if a model systematically undervalues or overvalues the prices observed for the warrants. A positive value indicates that the model overvalues the warrant market price, while a negative value indicates that the model undervalues the warrant market price. To decide what is the most efficient model we use the *Absolute Percentage Error*:

$$\text{Absolute Percentage Error} = \left|\frac{\text{model value - market price}}{\text{market price}}\right| \times 100\% \tag{3.20}$$

The methodology used is similar to other studies such as Noreen and Wolfson (1981), Ferri, Kremer and Oberhelman (1986), Schulz and Trautmann (1989 and 1994), Lauterbach and Schultz (1990), Stucki and Wasserfallen (1991),

[3] We are using different models. Therefore k = 1, 2, 3 or 4.

Veld (1992), Kremer and Roenfeldt (1992), Hauser and Lauterbach (1997), Shastri and Sirodom (1995) or Low (2000).

4. DATA

4.1. Characteristics of equity warrants traded on Euronext - Lisbon

4.1.1. Issuing and Listing Date

Up to 1999, the Portuguese law only allowed warrants to be issued when linked to bonds. After the issue, the warrants were typically split from the bond and listed as an independent financial asset.

The time period between the issue and the listing on the stock exchange varied from 1.71 months (in the case of Jerónimo Martins) to 12 months (in the case of Banco Comercial de Macau (BCM)). In average warrants took 4 months to be listed.

4.1.2. Exercise Ratio

We could not detect any particular pattern when observing the exercise ratio of the issues of warrants in Portugal. Four issues show a exercise ratio smaller than 1[4], four issues show a ratio greater than 1 and the remaining three have a ratio equal to 1. The smallest exercise ratio (0.3333) belongs to Efacec and Cofina, while the largest exercise ratio (4) belongs to Sonae Indústria.

The initial exercise ratio of Jerónimo Martins was equal to 1. However, after having decided a stock split that occurred on November 26, 1997, with a ratio of 2.5 shares for each existing share, the initial conditions had to be adjusted. In such circumstances, there are two alternatives: (i) to adjust the exercise price to 16.36 € (that is, 40.90 €/2.5); (ii) to adjust the exercise ratio. Jerónimo Martins decided to change the exercise ratio, but it did not use the ratio of 1 for 2.5. The holder that exercised the warrant would get one share (the initial right) plus a free share and a partial right corresponding to a half of a share. This equals a split in the warrants' exercise ratio from 1 to 2.5.

[4] This means that one needs more than one warrant to exercise.

4.1.3. Exercise Period

Most of the Portuguese equity warrants are Pseudo-American (Bermuda)[5], while the remaining ones are European. Six warrants could be exercised during first year following the issue. The warrants of BCM, Somague, Inparsa A and B, and Modelo Continente have a precise exercise time period, which is 1 year (for BCM) and 1 month for the remaining issues.

4.1.4. Expiration Date

By the ending of our data (December 31, 2000), the warrants BCM, Tertir, Efacec, Inparsa A and B, Sonae Indústria and Modelo Continente had already expired. The warrants Somague, Engil, Cofina and Jerónimo Martins were still listed in Euronext - Lisbon and were expected to continue up to 2003.

4.1.5. Moneyness Degree (S/X)

In most of the cases (6), the price of the underlying stock, S, is greater than the exercise price, X, on the day when they are listed, that is, the warrants are typically issued in-the-money. The moneyness ratio S/X varies between 0.52 (Engil) to 2.7 (Inparsa A and B). The average moneyness ratio of these issues was 1.31 (in-of-the-money). The warrants Inparsa A and B were the only issued quite deep in-the-money. Veld (1992) argues that when warrants are issued deep in-the-money, this tends to guarantee they will be sold signalling a firm's need of rising capital.

4.1.6. Initial time to maturity when listing

The initial time to maturity when listing (the remaining life of the warrant at the moment of the listing in the stock exchange) is the time period between the moment when warrants are listed and the expiration date. The first warrant issued in Portugal (Banco Comercial de Macau) was listed in 1991 and had a time to maturity of 1.69 years (the BCM warrant was admitted to listing only one year after the issue, jointly with a bond).

[5] Even showing these Pseudo-American characteristics, the Black-Scholes still holds since no dividends are paid when these warrants start to be American type.

The shortest maturity was 1.15 years, for the Modelo Continente warrant, and the longest maturity was 6.59 years, for the Jerónimo Martins' warrant. The average initial maturity of the Portuguese warrants is 3.41 years.

4.1.7. Trading Frequency

The equity warrants market in Portugal has been quite illiquid. The average trading frequency ratio is 47.09%. The most dramatic case was Tertir that traded only 14 days out of the 926 in which it was quoted (1.51%)!

TABLE 4.1

Equity Warrants traded on Euronext-Lisbon

a) All equity warrants in Portugal were jointly issued with bonds, with the exception of the issue made by Futebol Clube do Porto - Futebol SAD. However, this issue occurred under a new legal regime that authorised the issue of the warrants independently. After the issue, all the warrants were split from the bonds and listed, independently; b) Tertir issued in June of 1992, 1 500 000 bonds with attached warrants. These were listed on 26/03/1993. At the issuing date there were five possible quarters to exercise the warrants: the 3rd quarter of each year from 1992 to 1996. However, the exercise price was a stochastic variable with a floor of 4.99 € because it depends on the stock price path of the underlying security; c) The exercise ratio of Jerónimo Martins was changed from 1 to 2.5 shares; d) Each Engil warrant had two rights: one to be exercised on 0.425 shares of Engil during the month of August 2000, with an exercise price of 10.42 €, and another right to be exercised on 0.425 shares of Engil during the month of August 2003 with an exercise price of 12.06 €; e) Inparsa issued bonds with warrants in November 1998. Each bond had attached 2 callable warrants (A and B), each one of them giving an exercise right of 2 Inparsa shares with an exercise price of 7.48 € per share. The warrant A was expected to expire within 2 years, and the warrant B was expected to expire within 3 years. However, they were called just 4 months after being listed on February 1, 1999. On the April 27, 1999 the warrants A and B were merged into one single warrant Inparsa. The exercise period occurred between May 1 and 31, 1999 and the warrants stopped to be quoted on May 24, 1999.

Underlying Stock	*Number of warrants Issued*	*Issuing Date* [a]	*Listing Date*	*Exercise Ratio*	*Exercise Price*	*Exercise Period*	*Expiration Date*
BCM	1 875 000	22-10-90	22-10-91	1	14.96 €	01/07/92 to 30/06/93	30-06-1993
Tertir	1 500 000	26-06-92	26-03-93	1	[b]	Last quarter of 92, 93, 94, 95, 96	30-09-1996
Jer. Martins	2 281 761	23-12-96	13-02-97	1[c]	40.90 €	15/08 to 15/09 of 97 until 03	15-09-2003
Efacec	4 500 000	06-12-96	14-02-97	0.3333	5.32 €	Feb. and Aug. of 97, 98 and 99 and Nov. 99	12-11-1999
Sonae Indústr.	8 000 000	23-02-98	14-05-98	4	9.60 €	Nov. 98; May and Nov. 99-00	20-11-2000
Somague	10 000 000	05-05-98	03-09-98	0.5	12.47 €	14/05 to 16/06 of 2003	16-06-2003
Cofina	3 000 000	03-08-98	14-10-98	0.3333	17.46 €	During July 2001 and 2003	31-07-2003
Engil [d]	7 000 000	11-08-98	16-10-98	0.425	10.42 €	August 2000	31-08-2000
					12.06 €	August 2003	31-08-2003
Inparsa A [e]	10 000 000	05-11-98	01-02-99	2	7.48 €	December 2000	04-01-2001
Inparsa B [e]	10 000 000	05-11-98	01-02-99	2	7.48 €	December 2001	04-01-2002
Modelo Cont.	5 000 000	09-08-99	21-10-99	1.5	5.00 €	15/11 to 15/12 of 2000	15-12-2000

TABLE 4.2

Statistics of equity warrants when first listed

a) The Trading Frequency represents the percentage of days that the warrants were traded, relative to the number of trading days.

Underlying Stock	*Shares outstanding on listing date*	*Dilution Ratio (Mγ)/(Mγ+N)*	*Moneyness Degree S/X*	*Initial Time to Maturity (in years)*	*Trading Frequency* [a)]	*Type of Warrant*
BCM	6 500 000	22.39%	1.09	1.69	9.64%	Pseudo-American
Tertir	6 400 000	18.99%	-	3.52	1.51%	Pseudo-American
Jer. Martins	26 412 612	7.95%	1.17	6.59	27.47%	Pseudo-American
Efacec	10 131 580	12.75%	1.33	2.74	21.86%	Pseudo-American
Sonae Indústria	30 600 000	51.12%	1.45	2.52	85.78%	Pseudo-American
Somague	17 100 100	22.62%	0.65	4.79	52.78%	European
Cofina	5 000 000	16.67%	0.71	4.80	49.82%	Pseudo-American
Engil	16 100 600	15.60%	0.52	4.84	31.14%	Pseudo-American
Inparsa A	58 500 000	25.48%	2.70	1.92	100.00%	European
Inparsa B	58 500 000	25.48%	2.70	2.92	100.00%	European
Modelo Contin.	150 000 000	4.76%	0.77	1.15	37.94%	European
Average		**20.35%**	**1.31**	**3.41**	**47.09%**	

On the other extreme we have the Sonae Indústria's warrants that traded during 487 days out of the 574 in which they were quoted (85.85%). Another extreme case of liquidity was the Inparsa A and B warrants. They were traded every day but were called for exercise by the issuer, only 74 days after the issue.

Table 4.1 and Table 4.2 show all the relevant information for all the issues occurred in Portugal for equity warrants, including: amount issued, issuing date, listing date, exercise price, exercise period, expiration date, exercise ratio, exercise conditions, issue price, the first trading day, the dilution degree, the trading frequency, the moneyness degree, the time to the maturity and the warrant type.

Until 1998 there were few trades and consequently, we had few observations of warrant trading prices. Therefore we decided to select the time period between 1998 and 2000. During this time period, nine equity warrants were listed in Portugal. Two of the issues were on the same underlying firm (Inparsa) and they were issued together with the same bond. In other words, the buyer of one bond would get the bond plus two warrants Inparsa (A and B). Engil's warrant had two different exercise prices for two different periods, allowing the double exercise (exercise on both periods). That is, each warrant were in fact two warrants. This means that Engil's warrant was really a portfolio of two independent warrants, quite distinct from a typical warrant. For these reasons, we did consider neither the Inparsa A, Inparsa B nor Engil's warrants in our research. We used the remaining six listed equity warrants during 1998 to 2000. Table 4.3 describes and synthesizes the sample of the equity warrants.

TABLE 4.3

Characteristics of the sample of the equity warrants

Underlying Stock	*Observ.*	*Sampline Period*	*Date of the 1st observation*	*Warrants value*			*Average Time to Maturity (in years)*	*Dilution Ratio (mean)*
				Mean	*Minimum*	*Maximum*		
Jer. Martins	167	02-01-98 / 29-12-00	09-01-1998	52.73 €	23,99 €	101.00 €	4.80	6.78%
Efacec	36	02-01-98 / 10-12-99	12-06-1998	0.68 €	0,18 €	1.40 €	0.51	11.05%
Sonae Ind.	345	14-05-98 / 24-11-00	31-08-1998	1.95 €	0,01 €	14.49 €	0.83	43.28%
Somague	304	03-09-98 / 29-12-00	04-09-1998	0.23 €	0,02 €	0.51 €	3.54	22.62%
Cofina	122	14-10-98 / 29-12-00	16-10-1998	0.61 €	0,05 €	5.00 €	4.11	16.67%
Model Cont.	107	21-10-99 / 11-12-00	25-10-1999	0.96 €	0,01 €	2.30 €	0.55	4.76%
All	1 081			9.02 €	0,01 €	101.00 €	2.94	23.94%

We managed to collect a total of 1 481 observations, corresponding to 6 issues of warrants. From this set, 400 daily observations were excluded. For one of these observations, the underlying stock (Efacec) did not trade on that day, preventing the calculation of the implied volatility and, consequently, of the pricing models. All other 399 observations were excluded due to violations of the arbitrage conditions. The number of excluded observations in the sample means a substantial percentage on the starting sample.

To summarize, we selected 1 081 observations of the warrants daily closing prices, concerning to 6 issues of equity warrants, which, from now on, will be called the entire sample. Table 4.3 describes the sample characteristics, particularly the number of observations, sampling period, the date for the first observation, the warrants price, the time to maturity and dilution ratio.

5. EMPIRICAL RESULTS

As stated before, our aim was to analyse the impact of dilution and dividends on the goodness of fit of warrant pricing valuation models, to the Portuguese warrants market. We started by testing for violations to the arbitrage conditions. Then, the four models were calculated and the mean percentage errors were computed.

The greatest difficulty in our empirical study was the shortage of trustworthy data. Since the frequency trading of equity warrants in Portugal is very thin, we had to work with discontinuous time series and with very different time intervals in-between trades and closing prices (for instance, Jerónimo Martins traded infrequently and time gaps between two consecutive closing prices go from 1 to 102 days). Although in similar studies, authors reject these observations of thinly traded warrants, we kept them, firstly because one of our goals was to check whether thinly traded markets could show a different pattern of price behaviour and secondly because of the shortage of data.

Probably, as a result, our conclusions are somehow contradictory. But as we compare different models with the same dataset we may argue that this data shortage problem is common to all of them, which in terms of comparisons should not be significant.

5.1. Testing for Arbitrage Conditions

First we tested for violations on any of the arbitrage conditions. Table 5.1 shows the number of occurrences as well as their significance.

TABLE 5.1

Number of Observations Violating the Arbitrage Conditions[6]

represents the total number of observations for which the corresponding arbitrage condition was violated and % means its weight on the total number of observations per firm.

Warrant	$W \geq S - X$		$W \geq S - Xe^{-r(T-t)}$		$W \geq S_d - Xe^{-r(T-t)}$	
	#	%	#	%	#	%
Jerónimo Martins	0	0.0%	0	0.0%	0	0.0%
Efacec	38	43.7%	50	57.5%	25	28.7%
Sonae Indústria	121	22.3%	197	36.3%	197	36.3%
Somague	0	0.0%	0	0.0%	0	0.0%
Cofina	136	49.8%	150	54.9%	148	54.2%
Modelo Continente	0	0.0%	0	0.0%	0	0.0%
Total	295	19.9%	397	26.8%	370	25.0%

[6] Having that: $W_{american} \geq W_{bermudian} \geq W_{european}$, and $W_{european} \geq S_d - Xe^{-r(T-t)} \Leftrightarrow W_{european} \geq S - Xe^{-r(T-t)}$, then: $W_{bermudian} \geq S_d - Xe^{-r(T-t)}$ and $W_{bermudian} \geq S - Xe^{-r(T-t)}$

For three out of six warrants (Jerónimo Martins, Somague and Modelo Continente) we found no violations to any of the arbitrage conditions. The other three violate the 1st condition between 22% and 50% of the times, the 2nd condition between 36% and 58% of the times and the 3rd condition between 29% and 54% of the times. The 2nd condition shows the largest percentage of violations. When the stock price is adjusted to the dividends, S_d, the percentage of violations decreases. These very high percentages may lead us to infer for a serious warrant mispricing. However, non-synchronous data between the spot market and the warrant market may justify some of the violations of the boundary conditions. But there are also other reasons for this apparent high percentage in breaking boundary conditions (27% of the original sample). Warrants were relatively recent in the Portuguese market and, therefore, few investors were aware of warrant pricing theory; the market was very thin with public orders pending in the board for long time. Additionally we see no decrease in the number of violations, meaning that the market does not seem to have learnt with the passage of time. The boundary conditions were tested without considering any transaction costs.

5.2. Testing the Performance of Warrant Pricing Models

We measured the performance of models by the goodness of fit of the forecasted warrant price to market prices (assuming actual underlying stock prices, actual interest rates and recently averaged historical implied volatilities). When taken in relative terms we called it percentage error and percentage absolute error.

In order to avoid mixing any other effect, we compared the performance of the four valuation models according to different dimensions: moneyness degree, time to maturity and dividends. We also compared the models for the entire sample.

5.2.1. Testing Performance According to Moneyness Degree

We defined Moneyness degree as the ratio of the underlying stock price deducted by the net present value of the dividends paid until maturity, S_d, relative to the net present value of the exercise price, that is:

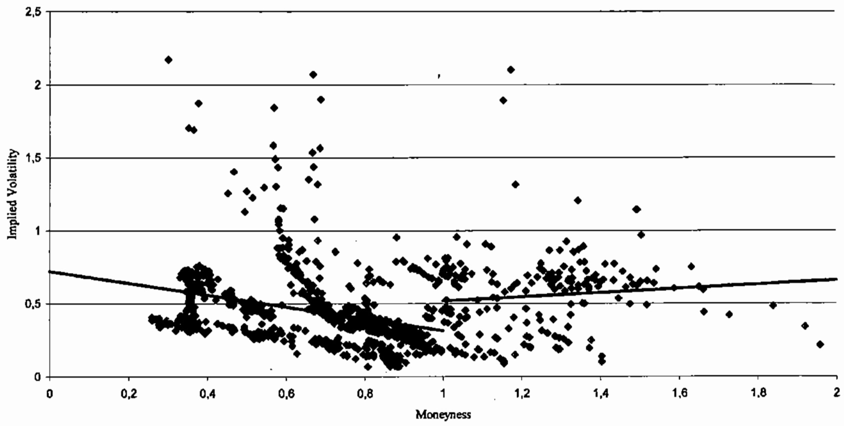
$$Mnss = \frac{S_d}{Xe^{-r(T-t)}} \tag{5.1}$$

According to the literature, implied volatility estimated from at-the-money stock options tends to differ from implied volatility of in-the-money and out-of-the-money options. Our treatment of the data follows approximately the same procedure to the one used by Duque and Lopes (2003) where the effect of the moneyness degree was tested as an explanatory factor of the differences found on implied volatilities.

We started by plotting implied volatilities estimated for each model for each of the warrant issued (please refer to Figures A.1 to A.6 in the appendix). We also plotted the entire sample in a single chart (Figure 5.1 bellow) in order to present a broad idea of any possible bias known as the smile effect. In a first glance we spot a significant number of observations with extremely high implied volatilities (well above 100%). However, we do not observe any of these high implied volatilities when we restrict the analysis to the at-the-money observation. We also observe visually a bias related to the moneyness degree (smile effect).

FIGURE 5.1

The Four Black and Scholes Implied Volatilities of the Entire Sample According to Moneynes



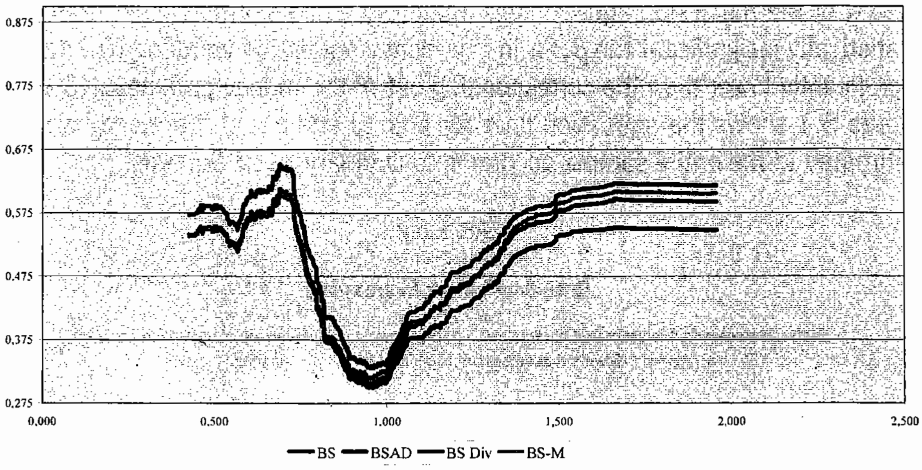
In some observations we observe some extremely high values for implied volatility figures, which may well represent strongly overpriced warrants.

The smile effect presented in the entire literature on option pricing bias is somehow more evident in Figure 5.2 after having sorted the entire sample by moneyness degree and, for each observation, having averaged the previous 200 observations.

From Figure 5.2 we draw two main conclusions: first, different models tend to show the same kind of pattern, although the Black and Scholes model adjusted for dilution (BSAD) seems to present systematically higher implied volatilities; second, the smile effect is asymmetric (out-of-the-money options seem to be far more sensitive to changes in moneyness than in-the-money options).

FIGURE 5.2

Implied Volatility Smiles Considering the Entire Sample



In order to test the statistical significance of the findings just presented we considered the following regression equations using only the Black-Scholes implied volatility dataset:

$$\sigma_{imp,in} = \beta_0 + \beta_1 (Mnss), \qquad \text{for Mnss} > 1 \qquad (5.2)$$

$$\sigma_{imp,out} = \beta_0 + \beta_1 (Mnss), \qquad \text{for Mnss} < 1 \qquad (5.3)$$

These regression equations were first calculated for each firm and next to the entire sample (Table 5.2).

As a very general comment we may say that implied volatilities in the Portuguese warrants market tend to consistently show the well-known smile effect, although they are more evident and statistically significant for out-of-the-money warrants. In Table 5.2 while all β_1 are statistically significant for out-of-the-money options, only for Cofina β_1 was statistically significant in the case of in-the-money options. This lack of significance for in-the-money regression equation parameters is not a consequence of the number of observations. From all the regression equations estimated in Table 5.2, Cofina is the case for which we got the smallest number of data points.

In addition, the away-from-the-money bias is more sensitive for out-of-the-money options than for in-the-money options, which is consistent with the findings of Duque and Lopes (2003) when studying equity options traded in LIFFE. The absolute value of β_1 is far higher for all the out-of-the-money regression equations than for the in-the-money regression equations. Therefore, the same change in the underlying stock price moving away from the money has a higher impact on the out-of-the-money options mispricing than on in-the-money options mispricing. A similar conclusion can be obtained by comparing the absolute value of the slopes of the regression equations estimated to entire sample. Apart from being statistically insignificant the β_1 for in-the-money regression equation (0.142) was significantly lower than the corresponding out-of-the-money parameter (0.406)[7]. Figure 5.1 shows the regression lines for both in and out-of-the-money warrants estimated to the entire sample according to Ordinary Least Squares.

TABLE 5.2

Results of Linear Regressions

Each cell represents the estimated parameters and the corresponding t-statistic of regression equation.
$\sigma_{imp,in} = \beta_0 + \beta_1(Mnss)$ * Means that values are significant at a 95% confidence level.

To calculate the implied volatility, we use $\sigma_t^k = \frac{\sum_{i=1}^{5} \sigma_{imp_{t-i}}^k}{5}$

	Mnss < 1 (*out-of-the-money*)				Mnss > 1 (*in-the-money*)			
	β_0	β_1	R^2 Adj.	N. Obs.	β_0	β_1	R^2 Adj.	N. Obs.
Jerónimo Martins	2.233* (26.207)	-1.733* (-16.884)	0.840	55	0.509 (4.805)	0.117 (1.382)	0.008	112
Cofina	0.626* (22.551)	(-0.559)* (-15.652)	0.711	100	-0.005 (-0.053)	0.186* (2.813)	0.248	22
Modelo Continente	2.677* (12.108)	-2.742* (-9.964)	0.481	107				
Efacec					0.451 (1.170)	0.187 (0.604)	-0.019	35
Sonae Indústria	1.663* (31.801)	-1.583* (-23.448)	0.633	319	0.375 (2.200)	-0.143 (-0.977)	-0.002	26
Somague	0.722* (21.849)	-0.610* (-7.896)	0.168	304				
Total Sample	0,721 (26.489)	-0,406* (-10.203)	0,104	886	0,377 (2.750)	0,142 (1.292)	0,003	195

Additionally we split the entire sample into three groups (in, at and out-of-the-money) instead of the two just presented (Table 5.3) in order to observe in more detail the asymmetry of the smile effect for the entire sample. However, given the short number of observations, we did not compute the regression equations for individual firms as previously done in Table 5.2.

[7] At a 90% confidence level the slopes of both regressions and their corresponding confidence intervals are the following: $|\beta_2^{ITM}| = +0.142 \quad \in [-0.040; +0.322]$ and $|\beta_2^{OTM}| = +0.142 \quad \in [+0.340; +0.471]$.

TABLE 5.3

Results of Linear Regression Applied to the Entire Sample

Each cell represents the estimated parameters and the corresponding t-statistic of regression equation $\sigma_{imp,in} = \beta_0 + \beta_1 (Mnss)$. * Means that values are significant at a 95% confidence level.

	Mnss < 0.7 (*Out-of-the-money*)	0.07 < Mnss < 1.3 (*At-the-money*)	Mnss > 1.3 (*In-the-money*)
β_0	0.732* (24.508)	0.693 (-2.030)	0,430 (1.958)
β_1	-0.426* (-9.262)	1.129 (3.254)	0,106 (0,646)
R^2 Adj.	0.096	0.062	-0.004
N. Obs.	798	145	138

The conclusions drawn based upon the figures of Table 5.2 are reinforced by the figures presented in Table 5.3. We underline the most significant conclusions. First, the u-shaped form of the smile obtained from Table 5.3 is clearer than the one extracted previously from Table 5.2. The slope for the in-the-money observations is now statistically significant as for out-of-the-money options and the difference between both betas is now more significant. Second, as we concluded from Table 5.2, the results presented in Table 5.3 also document that away-from-the-money bias is more sensitive for out-of-the-money options than for in-the-money options. Lastly, the v-shape obtained from the slopes of the regression equation presented in Table 5.3 is widen than the corresponding v-shape form obtained from Table 5.2[8].

Then we calculated the average implied volatilities but the conclusions did not change (see Table 5.4).

Generally speaking, average far-from-the-money implied volatilities are higher than at-the-money implied volatility figures whatever the model under use. However, when implied volatility is extracted from the original Black-Scholes model it is always smaller than when it is obtained by the others models. This was previously found by when composing Figure 5.2 and is now confirmed by averaging implied volatilities that were grouped by moneyness. Maybe opposing to what we could expect, if the average Black and Scholes' implied volatility figures are always lower than other models' implied volatilities, this may mean that when introducing new parameters in warrant price modelling we are introducing new sources of uncertainty, increasing the degree of residual risk that is observable in the only residual variable implicitly estimated.

[8] The comparison of the arctangent of the slopes (Table 5.2 and 5.3) may prove the following: $|\beta_2^{ITM}| = +0.142 \in [-0.040; +0.322]$; $|\beta_2^{OTM}| = +0.142 \in [+0.340; +0.471$.

TABLE 5.4

Comparison of the Percentage Error with the Moneyness Degree

The Average Implied Volatility was calculated from the implied volatilties extracted for each model under consideration. The Percentage Error and the Percentage Absolute Error are estimated according to the following expressions:

$$\text{Percentage Error} = \frac{\text{model value - market price}}{\text{market price}} \times 100\%\text{; Absolute Percentage Error} = \left|\frac{\text{model value - market price}}{\text{market price}}\right| \times 100\%$$

The number of observations droped to 1.051 because don´t have implied volatility to put in the first five observations of each warrants.

Warrant Pricing Model	*No. of Observ.*	*Average Implied Volatility*	*Percentage Error*		*Absolute Percentage Error*	
			Mean	*σ*	*Mean*	*σ*
In-the-Money						
Black-Scholes	128	57,0%	0,98%	16,2%	9,36%	13,2%
BSdiv	128	60,6%	0,66%	16,5%	9,68%	13,3%
BS-M	128	63,0%	0,30%	16,4%	9,82%	13,1%
BSAD	128	63,1%	0,33%	16,5%	9,91%	13,2%
At-the-Money						
Black-Scholes	140	42,2%	2,32%	19,9%	12,81%	15,3%
BSdiv	140	44,8%	2,07%	19,7%	12,80%	15,1%
BS-M	140	45,4%	2,09%	19,6%	12,94%	14,9%
BSAD	140	47,2%	2,36%	20,5%	13,59%	15,5%
Out-of-the-Money						
Black-Scholes	783	46,2%	-2,50%	34,9%	19,86%	28,8%
BSdiv	783	46,5%	-2,57%	34,8%	19,86%	28,6%
BS-M	783	46,8%	-2,37%	34,9%	19,77%	28,9%
BSAD	783	49,8%	-2,59%	34,2%	19,38%	28,3%

When the comparison between models is based on terms of the percentage error taking into account the moneyness degree, we found that warrants in-the-money tend to show better performances (the results are presented in Table 5.4). This could be seen as a general tendency for the models under study to price better in-the-money warrants than at or out-of-the-money warrants. This is somehow unexpected since the literature shows that models tend to perform better for at-the-money options. Therefore the conclusions should be carefully read. In this study (the methodology used is common to all the empirical studies carried out on warrant pricing) we estimate volatility to calculate the theoretical warrant price based on the most recent implied volatility estimates. For each day (observation) the volatility is calculated as the average of the previous 5-days implied volatilities. So, instead of speaking of models that are "better to price" we rather prefer to speak about models that are "less sensitive to changes in parameters" (in the present case we are talking about volatility). And in fact, the lambda of in-the-money options is significantly lower than the lambda of at-the-money options.

Table 5.4 also shows that out-of-the-money warrants present negative percentage error, while in-the-money or at-the-money warrants present positive percentage errors. This means that while out-of-the-money warrants tend to be undervalued, in-the-money and at-the-money warrants tend to be overvalued.

A last word should be addressed to the comparative performance of the models under study (Table 5.4). And it is clear that the performance of each model depends on the moneyness degree. For warrants in-the-money the best model seems to be the original Black-Scholes model, for warrants out-of-the-money the best model seems to be the BSAD, while for warrants at-the-money the best model seems to be the BSdiv[9]. However differences are not significant. The errors inside of each moneyness degree are very similar and the bigger difference between the different models is 0,79% in at-the-money.

5.2.2. *Testing Performance According to Time to Maturity*

Table 5.5 presents percentage errors and implied volatility figures when warrants are segregated by time to maturity. We found that as maturity approaches, models tend to decrease its performance, which is particularly poor for short term warrants (with less than two months to maturity). This decrease in terms of performance is also matched by the increase on implied volatilities, similar to what was found in Duque and Paxson (1997). But, although this seems to be the general pattern, there is a notorious difference for warrants maturing between 1 and 2 years. Those warrants are the best performers (with lower percentage and absolute percentage errors) and show a significantly lower implied volatility.

When trying to spot differences among models we observed that no model performs systematically better than the others. However, when observing the percentage absolute error the BSAD is the best performing model either for very short term warrants, either for longer term warrants. Nevertheless, there are never great differences among the models[10].

[9] The word "best" is used as a synonymous of most stable according to our previous explanation above.

[10] This waving behaviour may show a positive hope for mean reverting stochastic volatility models. However, this is out of the scope of this paper.

TABLE 5.5

Comparison of the percentage error with the Time to the Maturity

The Average Implied Volatility was calculated from the implied volatilties extracted for each model under consideration. TTM = Time to Maturity (0,167 years = 2 months). The Percentage Error and the Percentage Absolute Error are estimated according to the following expressions:

$$\text{Percentage Error} = \frac{\text{model value - market price}}{\text{market price}} \times 100\%\,; \quad \text{Absolute Percentage Error} = \left|\frac{\text{model value - market price}}{\text{market price}}\right| \times 100\%$$

			Percentage Error		*Absolute Percentage Error*	
Model	*No. of Observ.*	*Implied Volatility*	*Mean*	*σ*	*Mean*	*σ*
TTM > 2 years						
Black-Scholes	587	47,87%	-0,12%	25,8%	16,38%	19,9%
BSdiv	587	48,38%	-0,17%	25,8%	16,38%	19,9%
BS-M	587	50,20%	-0,17%	25,8%	16,36%	19,9%
BSAD	587	50,73%	-0,53%	25,1%	15,90%	19,4%
TTM 1 to 2 years						
Black-Scholes	121	27,85%	0,47%	19,5%	13,90%	14,2%
BSdiv	121	27,95%	0,38%	19,3%	13,82%	14,0%
BS-M	121	27,97%	0,34%	19,2%	13,78%	13,9%
BSAD	121	31,91%	0,61%	20,5%	15,06%	14,7%
TTM 0,167 to 1 year						
Black-Scholes	283	44,55%	-1,29%	22,8%	15,28%	17,2%
BSdiv	283	46,83%	-1,44%	23,0%	15,44%	17,2%
BS-M	283	45,48%	-1,23%	23,0%	15,37%	17,3%
BSAD	283	50,75%	-1,23%	23,0%	15,34%	17,2%
TTM < 0,167 years						
Black-Scholes	60	87,54%	-18,79%	78,2%	48,73%	63,9%
BSdiv	60	89,77%	-19,68%	77,1%	48,78%	62,9%
BS-M	60	87,78%	-18,66%	78,4%	48,72%	64,1%
BSAD	60	93,73%	-17,82%	77,0%	47,47%	63,2%

5.2.3. Testing Performance Considering Dividend Paying Firms

Only two out of six warrants were issued by dividend paying shares. Therefore, as two of the models were adjusted for dividends we restricted our analysis to those firms that paid dividends during the sampling time period. We dropped those non-dividend paying companies from our sample in order to compare the relative performance of the models under scope. Table 5.6 presents the results:

TABLE 5.6

Percentage Error of Warrants of Dividend Paying Firms

The Average Implied Volatility was calculated from the implied volatilities extracted for each model under consideration. The sample was reduced to firms that paid dividends during the time period of analysis. Values in brackets represent the t-values.
* Means that value are significant at a 99% confidence level. The Percentage Error and the Percentage Absolute Error are estimated according to the following expressions:

$$\text{Percentage Error} = \frac{\text{model value - market price}}{\text{market price}} \times 100\%\,; \quad \text{Absolute Percentage Error} = \left|\frac{\text{model value - market price}}{\text{market price}}\right| \times 100\%$$

			Percentage Error		Absolute Percentage Error	
Model	*No. of Observ.*	*Implied Volatility*	*Mean*	*σ*	*Mean*	*σ*
Black-Scholes	412	50,4%	0,52% (0,26)	40,4%	20,1%* (11,63)	35,1%
Bsdiv	412	53,1%	0,20% (0,10)	40,2%	20,2%* (11,80)	34,7%
BS-M	412	54,5%	0,48% (0,24)	40,4%	20,1%* (11,62)	35,1%
BSAD	412	54,4%	0,17% (0,09)	40,1%	20,1%* (11,53)	34,7%

We found that although the models performance is very similar, it is the BSAD that better performs particularly when using the percentage error for performance indicator. Therefore, when firms having warrants, pay dividends, it seems significant to account for it in warrant price and modelling. It is also interesting to spot that among the models that exclusively consider dividend paying adjustments, the discrete model seems to outperform the Merton (1973) model. This is logical since the Merton model is expected to perform better for stock indices or stocks that pay dividends several times along the year, which is not the case for Portuguese stocks. As a final remark we may say that adjusting for dilution and dividends (particularly the discrete model) seems to result in lower implied volatility variation.

5.2.4. Testing Performance with the Entire Sample

In this item we used the entire sample with no segregations to examine whether one of the models systematically out or underperforms the others. Tables 5.7 and 5.8 summarize the empirical results.

TABLE 5.7

Percentage Error of All the Warrants

The Average Implied Volatility was calculated from the implied volatilties extracted for each model under consideration. Values in brackets represent the t-values. * Means that values are significant at a 99% confidence level. The Percentage Error and the Percentage Absolute Error are estimated according to the following expressions:

$$\text{Percentage Error} = \frac{\text{model value - market price}}{\text{market price}} \times 100\%\text{; Absolute Percentage Error} = \left|\frac{\text{model value - market price}}{\text{market price}}\right| \times 100\%$$

			Percentage Error		*Absolute Percentage Error*	
Model	*No. of Observ.*	*Implied Volatility*	*Mean*	*σ*	*Mean*	*σ*
Black-Scholes	1.051	47,0%	-1,43% (-1,472)	31,6%	17,65%* (21,85)	26,2%
Bsdiv	1.051	48,0%	-1,56% (-1,610)	31,4%	17,69%* (22,05)	26,0%
BS-M	1.051	48,6%	-1,45% (-1,491)	31,6%	17,65%* (21,84)	26,2%
BSAD	1.051	51,1%	-1,57% (-1,639)	31,1%	17,45%* (21,99)	25,7%

The t-statistic for matched samples (Table 5.8) shows that differences among the means are not statistically significant for most of the cases (at 1% and 5% significance level). However, when the BSAD model is compared with other models, differences start to be significant. In Table 5.7 the BASD model is the model that shows lower Absolute Percentage Error and differences are strongly emphasised in Table 5.8. This means that we found empirical evidence for the dilution effect in the Portuguese equity warrants market that is consistent with the findings of Hauser and Lauterbach (1997) and Low (2000). Therefore, we strongly recommend for warrant price modelling the use of models that accommodate both dilution and dividend paying effect, particularly in the discrete form.

TABLE 5.8

Implied Volatility Differences Using two Models and Matched Samples

The Average Implied Volatility was calculated from the difference of implied volatilities extracted for a pair of models (under consideration). The Percentage Error and the Percentage Absolute Error are estimated according to the following expressions:

$$\text{Percentage Error} = \frac{\text{model value - market price}}{\text{market price}} \times 100\%\,; \quad \text{Absolute Percentage Error} = \left|\frac{\text{model value - market price}}{\text{market price}}\right| \times 100\%$$

	Percentage Error					*Percentage Absolute Error*				
	Mean	*σ*	*T-statistic*	*Df*	*Sig (2 tailed)*	*Mean*	*σ*	*T-statistic*	*Df*	*Sig (2 tailed)*
BS <-> BSDIV	0,128%	0,91%	4,584	1050	0,000	-0,039%	0,82%	-1,547	1050	0,122
BS <-> BS-M	0,019%	1,32%	0,456	1050	0,649	0,002%	1,28%	0,042	1050	0,967
BS <-> BSAD	0,138%	3,27%	1,366	1050	0,172	0,193%	2,91%	2,155	1050	0,031
BSDIV <-> BS-M	-0,109%	1,54%	-2,299	1050	0,022	0,041%	1,44%	0,917	1050	0,359
BSDIV <-> BSAD	0,010%	3,16%	0,101	1050	0,920	0,232%	2,81%	2,677	1050	0,008
BS-M <-> BSAD	0,119%	3,26%	1,187	1050	0,236	0,192%	2,90%	2,143	1050	0,032

6. CONCLUSIONS

Only recently the literature on equity warrants presented sophisticated methods to deal with dilution or dividend paying stocks. The main contributions to the topic have been Merton (1973), Roll (1977), Galai and Schneller (1978), Geske (1979, 1981), Whaley (1981), Lauterbach and Schultz (1990) or Schulz and Trautmann (1989 and 1994). It is expected that dilution and dividends have some impact on market prices for warrants. However, do we still notice analogous effects in illiquid markets? Thin trading introduces pricing bias that may well absorb all typical effects that we see in other warrant markets, namely the dilution and dividend effects. It could be interesting to check empirically whether dividends and dilution have some impact on warrants market prices, using a quite illiquid market as the Portuguese.

In order to avoid any other undesirable effects we developed our research exclusively within the Black-Scholes framework. We chose four warrant pricing models: the original Black-Scholes model and three of its derivations. Using these four models we empirically estimated values for actual warrant prices, and calculated the mean percentage error for each, as the difference between model prices and market prices. We assumed that the most efficient model shows the smallest percentage error. We used data supplied from the Euronext - Lisbon from 1998 to 2000.

Implied volatility extracted by the models, was higher for warrants in-the-money and out-of-the-money and lower for the warrants at-the-money, showing signs of the so called "smile effect". This effect seems not to be symmetric since we found that out-of-the-money warrants are more sensitive to the exercise bias than in-the-money warrants. Additionally, we found a strong pattern for the increase of implied volatility as time to maturity decreases. This resembles stock option markets and supports the findings of Duque and Lopes (2003).

Although there are no strong differences among the models the results lead us to conclude that the in the Portuguese warrants market from 1998 to 2000, the BSAD model, which accounts for the dilution effect and net present value of dividends, outperforms the others. These results are evident after comparing the models based on the Mean Absolute Error methodology. The BSAD model shows the lowest absolute percentage error and its standard deviation among all the models. When the comparison is between pairs of models, the results show irrelevant differences among them, except when comparing the BSAD model with every single one. In these cases the differences between each model and the BSAD model turns strongly significant, showing this model to be apparently the most appropriated model to price equity warrants, as expected by the theoretical literature.

Although this research strongly supports the appropriateness of the BSAD model to value equity warrants these results are still restricted by some limitations. Firstly, the equity warrants under scope are typically Bermudian style options that are not appropriately value by European style models that were used in this research. Secondly, the models used assumed dividends as known and certain. Stochastic dividend models should alternatively be used, although out of the scope of this paper.

APPENDIX

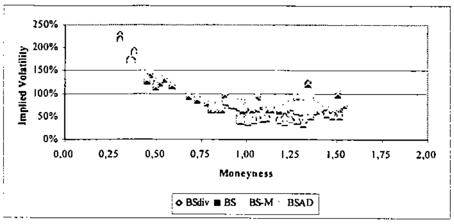


Figure A.1 - Implied Volatility versus Moneyness for Jerónimo Martins

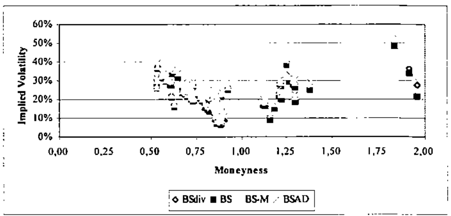


Figure A.2 - Implied Volatility versus Moneyness for Cofina

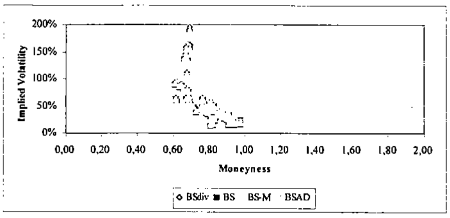


Figure A.3 - Implied Volatility versus Moneyness for Modelo Continente

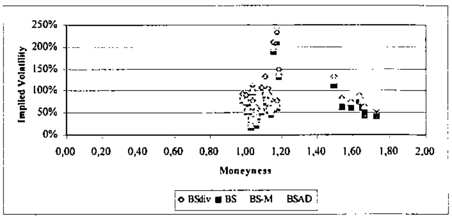


Figure A.4 - Implied Volatility versus Moneyness for Efacec

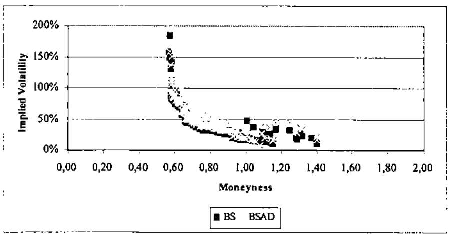


Figure A.5 - Implied Volatility versus Moneyness for Sonae Indústria

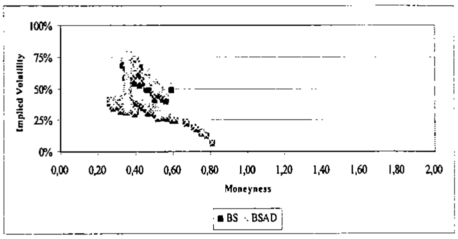


Figure A.6 - Implied Volatility versus Moneyness for Somague

References

Bierman, Harold Jr. (1973). The Cost of Warrants. *Journal of Financial and Quantitative Analysis*, *8*, 499-503.

Black, Fisher (1975). Fact and Fantasy in the Use of Options. *Financial Analysts Journal July-August*, 36-41, 61-76.

Black, Fisher and Scholes, Myron (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy*, *81*, 637-654.

Chen, A. H. Y. (1970). A Model of Warrant Pricing in Dynamic Market. *Journal of Finance*, *25*, 1041-1059.

Constantinides, George (1984). Warrant Exercise and Bond Conversion in Competitive Markets, *Journal of Financial Economics*, *13*, 371-397.

Cox, John and Rubinstein, Mark (1985). *Options Markets*. Prentice Hall, Englewood Cliffs, New Jersey.

Crouhy, Michael and Galai, Dan (1991). Warrant Valuation and Equity Volatility. In F.J. Fabozzi (ed.), *Advances in Futures and Options Research*, 5, 203-215.

Dravid, A.; Richardson, M. and Sun, T. (1993). Pricing Foreign Index Contingent Claims: An Application to Nikkei Index Warrants. *Journal of Derivatives*, *1*, 33-51.

Duque, João (1994). *The Meaning of Implied Volatility in Pricing Stock Options Traded in Options Markets*. PhD Thesis, Manchester Business School.

Duque, João and Lopes, Patrícia (2003). Maturity and Volatility Effects on Smiles: or Dying Smiling?. *Portuguese Economic Journal*, 2, 173-193.

Duque, João and Paxson, Dean (1997). Empirical Evidence on Volatility Estimators. Working paper 5/1997, Universidade Técnica de Lisboa, Instituto Superior de Economia e Gestão.

Emanuel, D. C. (1983). Warrant Valuation and Exercise Strategy. *Journal of Financial Economics*, *12*, 221-235.

Ferri, Michael, Kremer, J. W. and Oberhelman, H. D. (1986). An Analysis of the Pricing of Corporate Warrants. *Advances in Futures and Options Research*, *1*, Part A, 201-226.

Galai, Dan and Schneller, M. I., (1978). Pricing of Warrants and the Value of the Firm. *Journal of Finance*, *33*, 1333-1342.

Geske, Robert (1981). Comments on Whaley's Note. *Journal of Financial Economics*, *9*, 213-215.

Geske, Robert (1979). A note on an Analytical Valuation Formula for Unprotected American Call Options on Stocks with Known Dividends, *Journal of Financial Economics*, *7*, 375-380.

Hauser, Shmuel and Lauterbach, Beni (1996). Tests of Warrant Pricing Models: The Trading Profits Perspective. *Journal of Derivatives*, *4*, 71-79.

Hauser, Shmuel and Lauterbach, Beni (1997). The Relative Performance of Five Alternative Warrant Pricing Models. *Financial Analysts Journal*, *53*, 55-61.

Huang, Yu and Chen, Shing (2002). Warrants pricing: Stochastic volatility vs. Black–Scholes. *Pacific-Basin Finance Journal*, 10, 293-409.

Horst, Jenke and Veld, Chris (2003). Behavioral Preferences for Individual Securities: The Case for Call Warrants and Call Options. Working Paper. SSRN-id338420

Kim, Moon K. and Young, Allan (1980). Rewards and Risks from Warrant Hedging. *Journal of Portfolio Management*, *6*, 65.

Kremer, J. and Roenfeldt, Rodney (1992). Warrant Pricing: Jump-Diffusion vs. Black-Scholes. *Journal of financial and Quantitative Analysis*, 28, 255-272.

Lauterbach, Beni and Schultz, Paul (1990). Pricing Warrants: An Empirical Study of the Black-Scholes Model and its Alternatives. *Journal of Finance*, 45, 1181-1209.

Leonard, David and Solt, Michael (1990). On Using the Black-Scholes Model to Value Warrants. *Journal of Financial Research*, 13, 81-92.

Lim, Kian-Guan and Terry, Eric (2003). The Valuation of Multiple Stock Warrant. *Journal of Futures Markets*, 23, 517-534.

Loudon, Geoffrey and Nguyen, Kien (2006). Evidence on the issuer effect in warrant overpricing. *Applied Financial Economics*, 16, 223–232.

Low, Buen Sin (2000). Equity Warrant Pricing – Empirical Evidence from Malaysia KLSE Market, PHD Thesis, Manchester Business School.

Merton, Robert (1973). The Theory of Rational Option Pricing, *Bell Journal of Economics and Management Science*, 4, 141-183.

Noreen, Eric and Wolfson, Mark (1981). Equilibrium Warrant Pricing Models and Accounting for Executive Stock Options. *Journal of Accounting Research*, *19*, 384-398.

Reiner, Eric (1991). Quanto Mechanics, *Risk*, March, 59-63.

Samuelson, Paul (1965). Rational Theory of Warrant Pricing. *Industrial Management Review*, *6*, 13-31.

Schulz, G. Uwe and Trautmann, Siegfried (1989). Valuation of Warrants–Theory and Empirical Tests for Warrants Written on German Stocks. Working Paper, University of Stuttgart, October.

Schulz, G. Uwe and Trautmann, Siegfried (1994). Robustness of Option-Like Warrant Valuation. *Journal of Banking & Finance*, *18*, 841-859.

Shastri, Kuldeep and Sirodom, Kulpatra (1995). An Empirical Test of the BS and CSR Valuation Models for Warrants Listed in Thailand. *Pacific-Basin Finance Journal*, *3*, 465-483.

Sidenius, Jakob (1996). Warrant Pricing - Is Dilution a Delusion?. *Financial Analysts Journal*, September/October, 77-80.

Sisson, James R. (1987). The Evaluation of Employee Stock Options: An Inquiry into the use of Warrant Pricing Models. *Akron Business and Economic Review*, *18*, 25-47.

Spatt, Chester S. and Sterbenz, Frederic P. (1988). Warrant Exercise, Dividends, and Reinvestment Policy. *Journal of Finance*, *43*, 493-506.

Sprenkle, Case M., 1961, Warrant Prices as Indications of Expectations, Yale Economic Essays, 143-153; Reprint. in P. Cootner, ed. (1964). *The Random Character of Stock Market Prices*. MIT Press, Cambridge, Massachussets, 412-474.

Stucki, T. and Wasserfallen, W. (1991). Pricing of Warrants–An Empirical Evaluation, Discussion Paper, Study Center Gerzensee, Switzerland.

Ukhov, Andrey (2004). Warrant Pricing Using Observable Variables. *The Journal of Financial Research*, 27, 329–339.

Veld, Chris (1992). *Analysis of Equity Warrants: the Investment and Finance Instruments*. Tilburg University Press.

Veld, Chris (2003). Warrant Pricing: A Review of Empirical Research. *The European Journal of Finance*, *9*, 61-91.

Wei, Jason (1992). Pricing Nikkei Put Warrants. *Journal of Multinational Financial Management*, *2*, 45-75.

Wei, Jason (1995). Empirical Tests of the Pricing of Nikkei Put Warrants. *Financial Review*, *30*, 211-241.

Whaley, Robert (1981). On the Valuation of American Call Options on Stocks with Known Dividends. *Journal of Financial Economics*, *9*, 207-211.

Resumo

Este estudo tem como objectivo principal analisar o impacto da diluição e dos dividendos na adaptabilidade dos modelos de avaliação de warrants, aos dados do mercado português. A análise é baseada em dados da Euronext Lisbon, no período compreendido entre 1998 e 2000. De modo a evitar enviesamentos, na análise dos efeitos de diluição e dos dividendos, decidiu-se manter o estudo empírico apenas ao nível do Black-Scholes. Utilizaram-se quatro modelos de avaliação: o modelo original de Black-Scholes e três derivações. O estudo empírico a elaborar, consistiu em obter valores teóricos para os quatro modelos de avaliação e calcular um erro percentual médio para cada um deles em relação aos preços de mercado dos warrants. Constatou-se que o modelo original de Black-Scholes, quando ajustado à diluição e aos dividendos, proporciona uma performance superior na avaliação dos warrants portugueses para o período compreendido entre 1998 e 2000

Palavras-chave: Warrants, volatilidade implicita, Modelo de Black-Scholes, efeito de diluição, mercado português.

JEL Classifications: G13, G14