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MARKOWITZ EFFICIENT FRONTIER AND CAPITAL MARKET LINE – EVIDENCE FROM THE PORTUGUESE STOCK MARKET

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Abstract

This paper estimates the efficient frontier and the capital market line using listed stocks of the Portuguese capital market that are part of the PSI20 index, considering two different periods - before and after the 2008 financial crisis, known as the Global Financial Crisis. The results show the impact of the 2008 financial crisis on the global minimum variance portfolio and on the market portfolio. The sensitivity analysis of the results to the inclusion or not of the year 2008 is also considered[1].

Key words: Markowitz portfolio theory. Mean-variance theory. Efficient Frontier. Capital Market Line. PSI20.

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Introduction

This paper has the objective of determining the efficient frontier (EF)[2] and the capital market line (CML), using data from companies listed in the PSI 20, considering two different periods: the pre and post period of the financial crisis of 2008. Likewise, it will be possible to detect the changes in the delineation of the EF and the CML in times of crisis.

Under the assumption of economic rationality, investors choose to maintain efficient portfolios, i.e., portfolios that maximize the return for a certain level of risk, or that minimize the risk for a certain level of expected return, always considering investors' demands. According to Markowitz (1952, 1959), these efficient portfolios are included in the opportunity set of investments, which we can refer to as the set of all portfolios that can be constructed from a given set of risky assets.

While the classic portfolio theory states that assets selection is determined by the maximization of the expected value, the modern portfolio theory defends that the investment strategy must be conducted in a risk diversification perspective, which leads to an analysis of the combination of assets, considering return and risk, and also the covariance of the returns between assets. Mean-variance theory is a solution to the portfolio selection problem, which assumes that investors make rational decisions. In this context, portfolio construction is concerned with reducing portfolio concentration in order to improve its risk-return profile. Conceptually, the mean-variance analysis links diversification with the notion of efficiency, as optimal diversification is achieved along the efficient frontier.

Considering Markowitz modern portfolio theory, the optimal portfolio should be the tangency portfolio between the EF and the highest indifference curve, or, in other words, the efficient portfolio with maximum expected utility. Under the economic theory of choice, an investor chooses among the opportunities by specifying the indifference curves or utility function. These curves are constructed so that the investor is equally happy along the same curve, which leads to an analysis of the assumed investor's profile. Nevertheless, limitations of diversification are becoming more and more prominent (Amenc et al., 2011).

Tobin (1958) introduced leverage to portfolio theory by adding a risk-free rate asset. By combining this risk-free asset with a portfolio on the EF, one can construct portfolios with better outcomes that those simply on EF. This is represented by the capital market line, which is a tangent line from the risk-free asset until the risky assets region, i.e., the set of investment possibilities created by all combinations of the risky and riskless assets. The tangency point originated by the combination of the CML and the EF represents the searched market portfolio.

One of the main problems with this single-period theory on the choice of those portfolio weights that provide the optimal tradeoff between the mean and the variance of the portfolio return for a

[2] Also known as the risky assets efficient frontier or Markowitz efficient frontier.

future period is that the means, variances, and covariances of the underlying asset returns are unknown, and, in practice, are often estimated from historical data. As a consequence, this has led to the existence of portfolios that may perform poorly and have counter-intuitive asset allocation weights - referred to as the "Markowitz optimization enigma" (Michaud, 1989; Lai at al., 2009). In addition, few implementations of portfolio theory have been carried out in the literature, and the majority of portfolio managers do not use it. According to Kolm et al. (2014), the reasons for this is the existence of an irrational relation between inputs and outputs, as well as the great sensitivity of portfolio allocation to changes in inputs. The problems arise with the choices of the risky assets, the length of time, and the time period. Aware of this issue, this paper focusses attention on the estimation and sensitivity analysis of mean-variance-efficient portfolios to changes in input data of Portuguese listed companies included in the PSI20, estimated from historical data before, and after the 2008 crisis.

The paper is organized as follows. In the second section, we briefly present the theoretical framework and literature review regarding the EF, CML, and the choice of the optimal portfolio. In the third section, we carry out an analysis of the Portuguese stock market and of the PSI20 index. Data set and methodology used is discussed in the fourth section, and the results are presented in Section Five. Finally, the conclusion is presented in Section Six.

Theoretical Framework

In this section we review some basic concepts of modern portfolio theory, namely the efficient frontier and the capital market line, which have been discussed at length in the literature and in finance textbooks (Bodie et al., 2009; Elton et al., 2010).

The return of a financial security is the rate computed, based on what an investment generates during a certain period of time, where we include the capital gains/losses and the cash-flows that it may generate (i.e., dividends in the case of stocks). We calculate the returns by the difference between an asset price at the end and at the beginning of a selected period, divided by the price of the asset at the beginning of the selected period, $R_{it} = \frac{P_{it} - P_{it-1}}{P_{it}}$, where R_{it} is the return of asset i on moment t; P_{it} is the asset i price on moment t; and P_{it-1} is the asset i price on moment t-1. If t represents a week time interval, weekly returns data are obtained. In addition, the arithmetic mean of the return of the period is calculated as $\bar{R}_i = \frac{\sum_{t=1}^{T} R_{i,t}}{T}$, where T is the number of observations.

Risk represents uncertainty through the variability of future returns. Markowitz (1952, 1959) introduced the concept of risk, and assumed that risk is measured by the variance, σ_i^2, or by the standard deviation from the average return, given by $\sigma_i^2 = \frac{\sum_{t=1}^{T} \left(R_{i,t} - \bar{R}_i\right)^2}{T-1}$. Adopting the market model, it is possible to split risk into two types: systematic and non-systematic (Sharpe, 1965; Banz, 1981; Fama and French, 1992, 1993, 2004)). Non-systematic risk is the part of risk that cannot be associated with the behaviour of the economy, i.e., it depends exclusively on the asset's characteristics, and it is a function of variables that affect the company's performance. It is a kind of risk that can be eliminated by the diversification process in the construction of a portfolio. Systematic risk is connected with the fluctuations of the economic system as a whole. This type of risk cannot be eliminated through diversification, as it is governed by market behaviour.

Covariance enhances the influence of an asset on other asset with different characteristics in the determination of the variance of a portfolio. It measures how returns on assets move together. In other words, a positive covariance shows that when an asset return is positive, then the other considered asset tends to also have a positive return, and the reverse is also true. A negative covariance shows that the rates of return of two assets are moving in the opposite directions, i.e., when the return on a certain asset is positive, the return on the other considered asset tends to be negative, which is true for the inverse situation. The case where two assets have zero covariance means that there is no relationship between the rates of return of the considered assets. In this context, the correlation is a simple measure used to standardize covariance, scaling it with a range of -1 to +1.

The covariance concept was first developed by Markowitz (1952), who referred to the importance of the diversification on the choice of the optimal portfolio. After the introduction of the modern portfolio theory, investment strategy started to be conducted with a risk diversification perspective, meaning that, for each level of risk, a combination of assets exists, which leads to at least the same return and a lower level of risk. In turn, this leads to the representation of the efficient frontier, where each level of return has the minimum risk.

Another theory underlying this concept is the mean-variance theory, which shows up as a solution for the portfolio selection problem. Markowitz (1952) demonstrates that the expected return of a portfolio is based on the mean of the assets' expected returns, while standard deviation is not simply a mean of the individual assets standard deviation, but also considers the covariance between the assets' returns. Markowitz (1952, 1959) assumed that, for each investor, it is possible to select a portfolio that attains investors' expectations, which is called the efficient frontier in the portfolio possibilities curve. In this context, all investors always want to maximize the expected return, given a determined level of risk, or to minimize the risk, given a determined level of expected return. Implicitly, investors are risk-averse and assume the mean-variance theory for selection criterion, i.e., the mean and the standard deviation of the returns (Brennan and Lo, 2011). Markowitz (1952, 1959) also assumed that we are in a situation of perfect markets, meaning that there are no transaction costs, neither taxes, and that assets are endlessly indivisible.

Therefore, assuming economic rationality, all investors choose to have efficient portfolios. The selected portfolio must be above the global minimum variance portfolio, or in the concave portion of the portfolio possibility curve. The optimum portfolio will always depend on investors' preferences, mainly their risk aversion profile, which can be characterized by the utility function.

This is based on the economic theory of choice, where an investor chooses among opportunities by specifying a series of curves which are called indifference curves. These so called curves are constructed so that the investor is assumed to be equally satisfied everywhere, along the same curve. In this way, for each investor, the optimal portfolio should be the tangency portfolio between the efficient frontier and the indifference curve, i.e., the efficient portfolio with greater expected utility from the investor's perspective (Markowitz, 1987).

Formally, the portfolio expected return, with N risky assets, is the weighted average of the expected returns of the single assets that comprise the portfolio, which can be represented as $\overline{R}_p = \sum_{i=1}^{N} w_i \bar{R}_i$, where N is the number of assets in the portfolio, w is the weight of each asset in the portfolio, and $\bar{R}_i$ is the average return of asset I, which, assuming equal weights, yields $\overline{R}_p = \frac{\sum_{i=1}^{N} R_i}{N}$.

Variance is simply the expected value of the squared deviations of the return of the portfolio from the mean return of the portfolio, that is to say, $\sigma_p^2 = E\ (R_p - \bar{R}_p)^2$. The standard deviation is represented by the squared root of the variance, $\sigma_p = \sqrt{\sigma_p^2}$, which is the measure assumed as the asset risk for a certain period of time.

Substituting in this general formula the expression for return on a portfolio and expected return on a portfolio, yields $\sigma_p^2 = \sum_{i=1}^{N}(w_i^2 \sigma_i^2) + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} (w_i w_j \sigma_{ij})$, where σ_i^2 is the variance of the return on asset i, and σ_{ij} is the covariance between the returns on asset i and asset j, which is given by $\sigma_{i,j} = \frac{1}{T} \sum_{t=1}^{T} (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j)\ , i \neq j$. Hence, portfolio variance is the sum of individual assets variances, plus the covariances between them, considering the weight of each asset in the portfolio.

The effect from diversification is obtainable by increasing the number of assets, which significantly reduces the value of the first term, tending to zero, and the value of the second term, tending to the average covariance. However, diversification is not able to eliminate the total risk of a portfolio, as returns of securities are not perfectly (negatively or positively) correlated, that is to say, $-1 < \rho < 1$, where $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$.

In conclusion, in order to reduce the total risk of a portfolio, an investor should diversify it. The higher the payoff from diversification, the lower (closer to -1) is the correlation coefficient between assets. The main goal of any investor is to decrease total risk, without affecting a certain desired level of return, and portfolio diversification is the key to solving this problem. We can define this as a portfolio strategy which allows investors to reduce exposure to risk by combining a certain amount of different assets (these can be stocks, bonds, futures, etc.).

The efficient frontier consists of a set of efficient portfolios, which are portfolios that have the highest return for a certain level of risk. The efficient frontier coincides with the top portion of the minimum-variance portfolio set. The portfolio with the lowest risk is known as the global minimum-variance (GMV) portfolio, considering that for each level of expected portfolio return, we can vary the portfolio's weights of individual assets to determine the minimum-variance portfolio, i.e., the one with the lowest risk (Feldman and Reisman, 2001). Any portfolio below the EF can be considered to be inefficient, i.e., we should not invest in it. This concept appears with the question of which combination of assets is the best one to opt for. Assuming investors are risk-averse, they prefer a portfolio that has the greatest expected return when choosing among portfolios that all have the same risk. The optimal portfolio[3] along the EF is selected, taking into consideration the investor's utility function and attitude towards risk (Merton, 1969; Elton et al., 2011; Girard and Ferreira, 2005).

Tobin (1958) introduced the risk-free asset for the analysis, with the development of the Tobin Separation Theorem (TSB). Hence, the problem of finding an optimal portfolio for a given level of risk tolerance can be separated into two easier problems: first finding an optimal mix of market securities that does not vary with risk tolerance, and then combining it with an appropriate amount of the risk-free assets (Buiter, 2003). Dybvig and Ingersoll (1982) prove that TBS can only be obtained if all investors have quadratic utility, and that the relation can only hold if arbitrage opportunities exist in the market.

The CML takes into account the inclusion of a risk-free asset in the portfolio. The CML is the tangent line drawn from the point of the risk-free asset to the feasible region for risky assets. The tangency point (usually M) represents the market portfolio, so-named as all rational investors (minimum variance criterion) should hold their risky assets in the same proportions as their weights in the market portfolio. Therefore, the CML is considered to be superior to the efficient frontier theorem, as all points along the CML have superior risk-return profiles to any portfolio on the efficient frontier, with the exception of the market portfolio, where the point on the efficient frontier forthe CML is the tangent. This portfolio is based entirely of risky asset and the market, and it has no holding of risk-free assets, i.e., money is neither invested in, nor borrowed from the money market account.

The CML represents the possible combinations of the market portfolio and risk-free asset. Similarly, the CML is defined as a risk-return trade-off derived by combining the market portfolio with risk-free borrowing and lending, with all portfolios between the risk-free and the tangency point being considered to be efficient. Formally, $E(R_p) = R_f + \frac{E(R_M)-R_f}{\sigma_M}\sigma_p$. This represents a linear function, where the slope is considered to be the compensation in terms of expected return for each additional unit of risk and the intercept point is the risk-free rate. Thus, the expected return of any portfolio on the CML is equal to the sum of risk-free rate and risk premium, where

[3] Or optimum portfolio.

the risk premium is the product of the market price of risk and the risk of portfolio under consideration.

As mentioned, assuming that all investors are rational and have certain preferences over a chosen set of assets, they will want to maximize their utility function, subject to their budget constraint. Therefore, the final decision for the determination of the optimal solution for the investor involves maximizing their expected utility, through the indifference curves. The optimal portfolio for each investor is obtained through the highest indifference curve that is tangent to the CML.

Alternatively, it is possible to choose a portfolio using performance measures that enable one to rank portfolios. Although it is beyond the scope of this paper to describe in detail these measures, it important to mention the Sharpe ratio, the Treynor ratio, and the Jensen ratio. Jagric (2007) explains that, years ago, investors were almost exclusively interested in having large returns, but in recent years, investors have started to look at the assets and portfolio performance, which also leads to a risk performance consideration. Levisauskait (2010) notes that if a portfolio is well diversified, then all these measures will obtain the same ranking of the portfolios, as well-diversified total variance is equal to the risk of the market (β=1). If this does not hold, then Treynor and Jensen measures can rank relatively undiversified portfolios much better than the Sharpe ratio does, as it uses both systematic and non-systematic risk.

Roy (1952) was the first to suggest a risk-to-reward ratio for evaluating a strategy's performance, and Sharpe (1966, 1975, 1994) introduced a measure for this performance analysis, applied to the Markowitz mean-variance theory. Formally, this measure is the $Sharpe\ ratio = \frac{R_p - R_f}{\sigma_p}$. Dowd (2000) concluded that the Sharpe ratio is a good measure, as it evalkuates both risk and return in a single measure, and, for example, an increase in return differential or a fall in standard deviation leads to a rise in this measure, which is considered to be a "good event". When we have to choose between several alternatives, this measure allows us to choose the one with the higher ratio. A negative value indicates that a risk-free asset would have a better performance that the actual portfolio. Important to note is the fact that all of the portfolios on the CML have the same Sharpe ratio as that of the market portfolio, i.e. $Sharpe\ ratio = \frac{R_p - R_f}{\sigma_p} = \frac{R_M - R_f}{\sigma_M}$. In fact, the slope of the CML is the Sharpe ratio of the market portfolio. Therefore, a stock picking rule of thumb is frequently used to buy assets whose Sharpe ratio is above the CML, and to sell those whose Sharpe ratio is below the CML. From the efficient market hypothesis perspective (Fama, 1965, 1973, 1991), it follows that it is impossible to beat the market. This abnormal extra return over the market return at a given level of risk is what is called the 'alpha'.

We can also measure performance by using the Treynor ratio (Treynor, 1965; Treynor and Black, 1973), which is a measure of excess return per unit of risk, i.e., it compares portfolio premium risk with the diversifiable risk of the portfolio measures by its beta, or by Jensen alpha (Jensen,1967), which measures the performance of an investment as a deviation from the state of equilibrium, based on the Capital Asset Pricing Model (CAPM) developed by Sharpe (1964) and Lintner (1965). It also measures the difference between an asset's actual return and the return that could have been made on a benchmark portfolio with the same beta. Merton (1972) derives the efficient portfolio frontiers for more than three assets, and verifies the characteristics of these frontiers.

There are many different techniques that can be used to deduce the EF (Elton et al., 1977; Bawa et al., 1979). The use of the single index model (SIM)[4] allows the reduction of the necessary inputs to the Markowitz portfolio selection and is considered to be an accurate description of reality. This model assumes that correlation between each security return is explained by a unique common factor, which is the rate of return on a market index, such as the S&P 500.

Historical data is usually used to obtain estimates of the inputs for the portfolio selection process, although analysts might modify these historical estimates to better reflect beliefs about the future. Therefore, efficient frontier technology is highly sensitive to input data estimates, with consequences for asset allocation decisions. On the other hand, different risky assets result in different draws of the EF and the CML, which means that there is no unique EF and CML, but as many as the number of possible risky assets sets one might consider to represent the market. Moreover, the behavioural finance literature provides evidence of the excess volatility in the aggregate stock market relative to the present value implied by the efficient markets model (Shiller, 2003; Kirman, 2010; Chatelain and Ralf, 2012), which questions the possibility of estimating both the EF and the CML.

As far as we are aware, this paper is a first attempt to build and delineate the EF and the CML using risky assets included in the PSI 20 Index for two periods, before and after the 2008 crisis.

Brief analysis of the Portuguese Stock Index

Portugal may be included in the group of bank-oriented countries with a universal bank system, which is strongly concentrated in a few financial groups, which means that money flows essentially through financial institutions (Allen and Gale, 2000; Garcia and Guerreiro, 2016). Banks and government dominate as a source of financing, but have not been immune to the financial crisis that started with the bankruptcy of the Lehman Brothers, in September of 2008 (Nanto, 2009). In addition, Portugal, like other European countries, had a financial assistance programme to solve their debt and economic structural problems. Hence, bank funding conditions were affected by sovereign credit risk (BIS, 2011).

Considering the capital market, the Portuguese Stock Index (PSI20) is the national stock index benchmark, constituted by the 20 biggest companies listed on the Lisbon Stock Exchange (Euronext Lisbon), given that the Portuguese capital market is small. The liquidity of each listed company is measured by the transaction volume in the stock exchange. The supervisory institution, the CMVM,

[4] Also denoted a market model.

requires financial reporting as being the available information needed for investors. According to the regulations of the CMVM, short sales are allowed, but there are some restrictions regarding these operations. Hence, considering that, we thus assume that short selling is not possible.

The base value of the PSI20 was 3,000 points, which started on December 31, 1992.

During the first week of May 2015, the PSI20 index included only 18 listed companies (Altri, Banco BPI, BANIF, BCP, CTT, EDP, EDP Renováveis, GALP Energia, Impresa, Jerónimo Martins, Mota-Engil, NOS, Portucel, Portugal Telecom, REN, Semapa, Sonae SGPS and Teixeira Duarte). Therefore, the empirical work considers of 20 risky assets, for the first period (from May 2000 to the end of 2008), and of 18 risky assets for the second period (from January 2009 to May 2015) to derive the EF and the CML.

Figure 1 shows weekly prices of the PSI 20 index for the entire period, from May 2000 to May 2015.

Fig. 1 - Evolution of the PSI20 price, 2000-2015

A similar behaviour, a significant decrease, is detectable around three events: the internet bubble in 2000, September 11th in 2001, and the Lehman Brothers collapse in 2008.

Data Set and Methodology

Historical data of weekly prices of all the stocks included in PSI20 were taken from the Datastream platform, from the first week of May 2000 to the first week of May 2015. We then split it into two different time periods to derive the EF of those periods. The first period is from the first week of May 2000 to the last week of 2008 (pre 2009), and the second period is from the first week of 2009 to the first week of May 2015 (post 2009). As previously mentioned, there is no ideal time period horizon for obtaining reliable input data (average returns, standard deviations and covariances), and therefore we considered periods between 5 to 10 years. In addition, an alternative first period was considered to disentangle the impact of the year 2008. Therefore, the sensitivity of the results was analysed with the exclusion of the year 2008 in the first period.

Table 1 presents the descriptive statistics for the periods under analysis.

Asset	January 2009 – May 2015		Asset	May 2000 - December 2008		May 2000 - December 2007	
	$\bar{R}_y$ (%)	$\sigma_{i,y}$ (%)		$\bar{R}_y$(%)	$\sigma_{i,y}$ (%)	$\bar{R}_y$ (%)	$\sigma_{i,y}$ (%)
Altri	15.80	34.22	Altri	24.80	45.54	46.46	42.45
Banco BPI	-10.37	46.77	Banco BPI	-10.94	27.89	3.76	21.98
BANIF	-48.89	132.26	BCP	-20.60	30.08	-8.06	24.63
BCP	-26.38	51.31	BES	-7.61	21.36	3.59	14.70
CTT	26.61	41.29	Brisa	1.62	20.56	11.32	16.66
EDP	1.73	25.87	Cimpor	-2.61	27.06	6.03	23.04
EDP Renováveis	-0.60	28.94	EDP	-7.07	26.48	0.03	22.04
GALP Energia	3.04	31.20	EDP Renováveis	5.34	37.95	-	-
Impresa	-9.29	45.44	GALP Energia	11.25	42.00	30.38	38.59
Jerónimo Martins	14.86	30.78	Jerónimo Martins	-3.61	35.98	2.01	31.27
Mota-Engil	-5.01	42.88	Mota-Engil	-1.67	29.21	10.76	25.82
NOS	4.31	31.97	NOS	-26.93	37.46	-19.45	35.68
Portucel	13.76	23.90	Portucel	-2.91	25.28	3.21	22.93
Portugal			Portugal	-9.64	31.26	-5.02	28.69
Telecom	-37.73	42.10	Telecom				
REN	-1.52	17.63	REN	10.80	36.88	17.19	37.70
Semapa	8.13	26.46	Semapa	3.21	25.73	8.58	25.33
Sonae SGPS	12.71	30.98	Sonae SGPS	-18.20	37.03	-2.23	33.52
Teixeira Duarte	-1.24	58.51	Sonaecom	-22.10	53.46	-10.28	53.56
			Sonae Industria	-20.58	33.77	-4,78	29,64
			Teixeira Duarte	-16.74	38.17	-1,07	30,66
PSI20	-2.94	22.67	PSI20	-9.04	18.90	-0.41	15.40
Risk free rate	0.169		Risk free rate	3.03		4.03	

Table 1 - PSI 20 stocks: means and standard deviations, 2009-2015, 2000-2008, 2000-2007

The EURIBOR rate for one year is used as the risk-free rate of return proxy for the calculations. The registered average values are 3.025% for the first period, and 0.169% for the second one.

Figures 2 shows annual means and standard deviations for the stocks included in the PSI 20 for the first period. 14 out of 20 stocks registered negative annual means, with standard deviations higher than 20%. Figure 3 shows annual means and standard deviations for the stocks included in the PSI 20 for the second period. In this period, nine out of 18 stocks registered negative annual means, with standard deviations more than 20% - a relative recovery in comparison with the previous period.

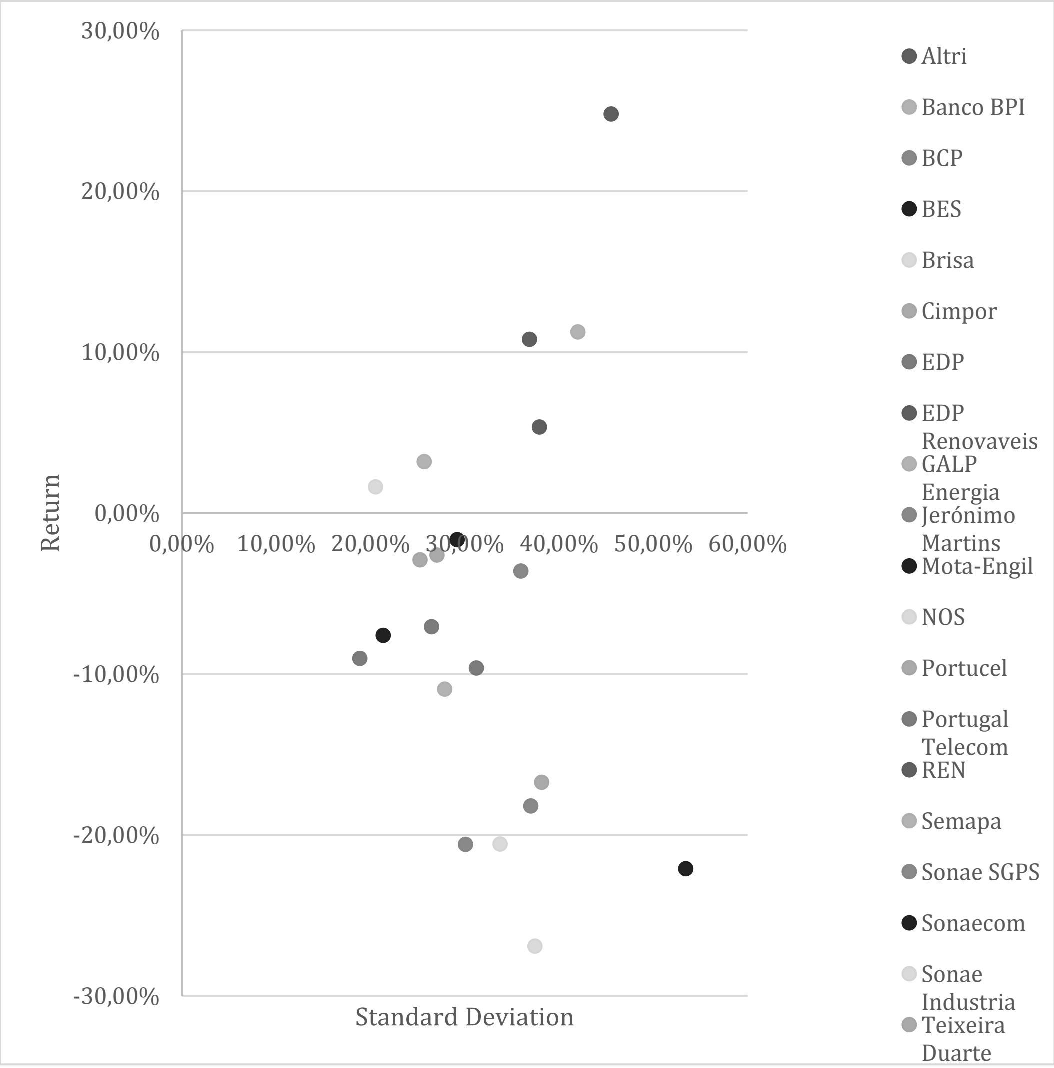

Fig. 2 - Risk and return of PSI 20 stocks, 2000-2008

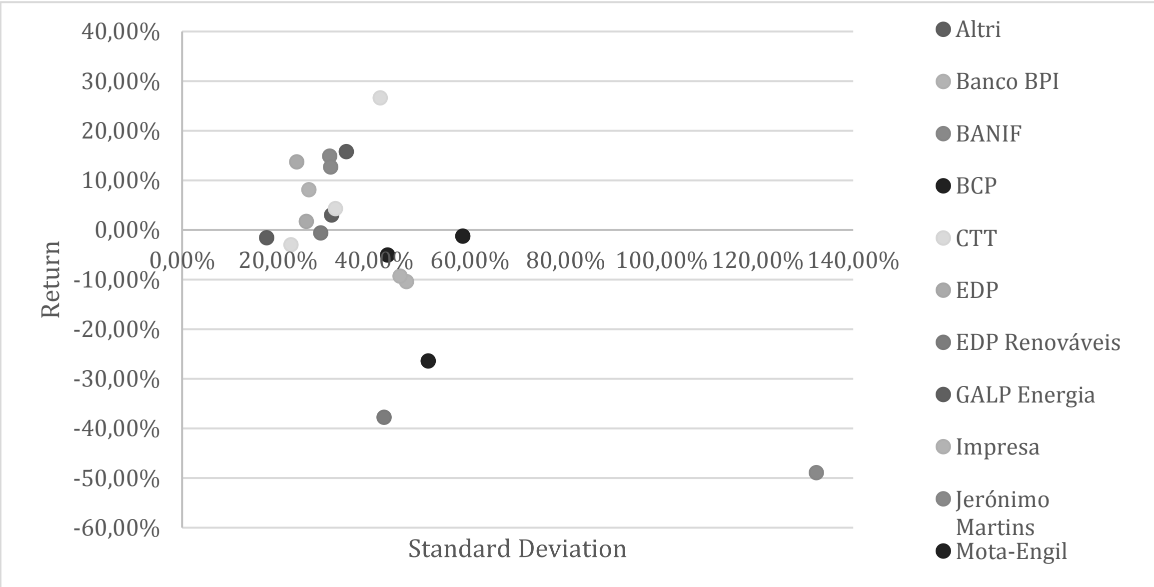

Fig. 3 - Risk and return of PSI 20 stocks, 2009-2015

The first period considers weekly returns of the 20 companies included in PSI20, and of the PSI20 index. This represents a sample of 451 observations, for each time series. The second period considers weekly returns of the 18 companies included in the PSI20, and of the PSI20 index, representing a sample of 333 weekly observations, for each time series. The alternative first period, comprises 19 companies and 399 observations for each time series. Thus, the weekly return rates were calculated, as well as the means, variances and covariances, for each stock i (i=1,…,N) and the PSI20. Historical means and standard deviations of each weekly time series were then annualised through the transformations $\bar{R}_{i,y} = (\bar{R}_{i,w} + 1)^{52} - 1$, where $\bar{R}_y$ represents the average annual return, and $\bar{R}_w$ is the weekly average return, and $\sigma_{i,y} = \sigma_{i,w} \times \sqrt{52}$, where $\sigma_{i,y}$ represents the annual standard deviation, and $\sigma_{i,w}$ is the the standard deviation of weekly returns.

The variance-covariance matrix for each period was obtained (Tables A1, A2 and A3 in the Appendix), displaying mostly positive imperfect correlations.

Once the input data were estimated, the construction of the EF and CML followed. To obtain efficient portfolios, we have to minimize the total risk of the portfolio, given by $\sigma_p^2 = \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} w_i w_j \sigma_{ij}$, for a given level of expected return (Markowitz, 1952). In order to align with the Portuguese **s**tock market features, some restrictions were assumed. No borrowing or lending is allowed, as the objective is to maximize the objective function $\theta = \frac{\bar{R}_p - R_f}{\sigma_p}$, subject to the constraint$\sum_{i=1}^{N} w_i = 100\%$. In addition, investors are not allowed to short-sell, which means that all assets have positive or zero investment, $w_i \geq 0, i = 1, \ldots, N$. Finally, in order to avoid the complete domination of only one asset in our portfolio, and to achieve Markowitz diversification, no asset is allowed to have a weight higher than 10% in the final portfolio, that is to say, is $w_i \leq 10$. This assumption is further specified in the results, with evidence that without this assumption, efficient portfolios could be composed of a single stock.

The Solver function was used to create several portfolios with different average return rates and variances, from the moment we achieved a minor portfolio return, up to the moment of a maximum possible return (Tables A4, A5 and A6 in the Appendix). This leads to the construction of the efficient frontier, considering the relation between the returns and the standard deviations of the "solved" portfolios. The CML was derived through the consideration of the equation $E(R_p) = R_f + \frac{E(R_M)-R_f}{\sigma_M}\sigma_p$. This represents a linear function from the risk-free asset rate of return up to the point of the market portfolio rate. Therefore, the CML is derived by drawing a tangent line from the intercept point on the efficient frontier to the point where the expected return equals the risk-free rate of return.

Results

The estimation results of the EF are depicted in Figure 4, for the first and the second periods considered. Clearly, it is possible to observe a huge change in the configuration of the EF between the two periods, with a movement to the top-right. In the period post-2009, the GMV portfolio registers a rate of return of 4.24%, which compares with the negative rate of return of -3.28% of the GMV portfolio obtained for the pre-2009 period.

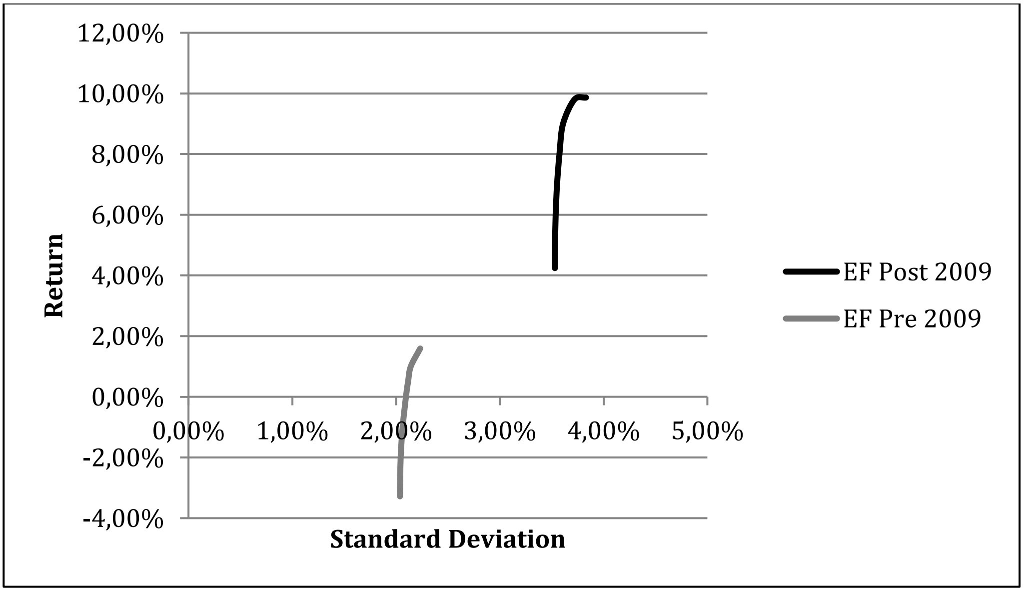

Fig. 4 - Efficient Frontiers, 2000-2008 and 2009-2015

The maximization of the Sharpe ratio leads to the determination of the CML. In fact, the slope of the CML is the Sharpe ratio of the market portfolio. The CML results from the combination of the risk-free asset and the market portfolio. All points along the CML have superior risk-return profiles to any portfolio on the efficient frontier, with the exception of the market portfolio, which is the point on the efficient frontier where the CML is the tangent. Figure 5 depicts the results for the CML derived from the existent EF, for both periods.

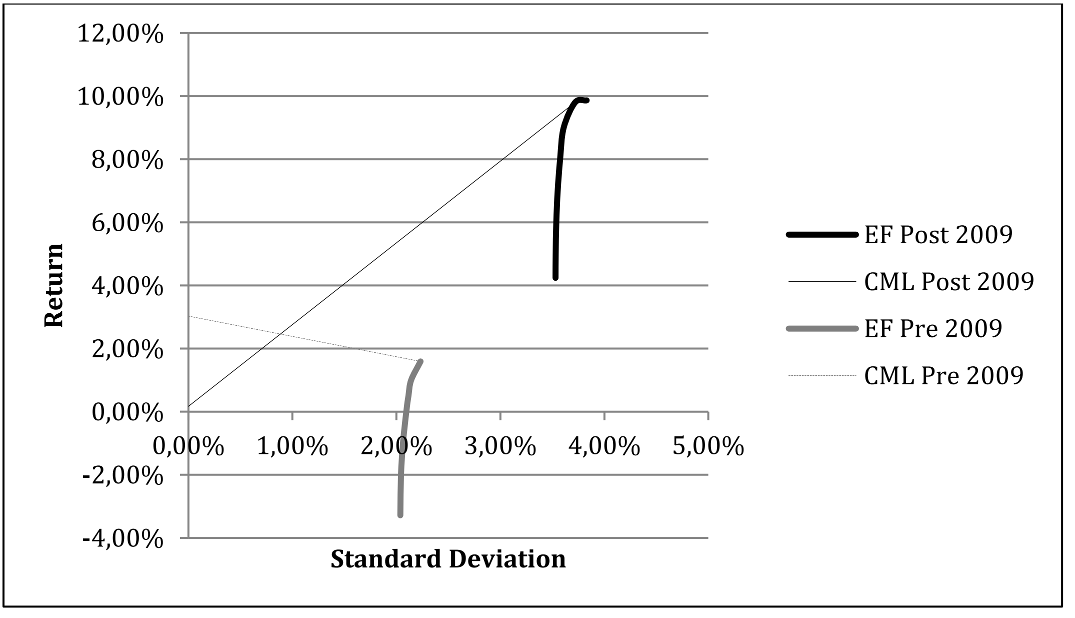

Fig. 5 - Capital Market Lines, 2000-2008 and 2009-2015

The tangent portfolios present a rate of return of 9.80%, for the post-2009 period, and a rate of return of 1.59%, for the pre-2009 period. Oddly, the tangent portfolio of the first period registers a rate of return lower than the risk-free rate. Consequently, the estimated CML presents a negative slope, contrasting with the concept of an efficient portfolio. Indeed, independently of investor preferences, the optimal portfolio would be totally composed of the risk-free asset.

In addition, with regards to the second period, if the weights restriction was relaxed, then the portfolio with 100% invested in the CTT asset registers the higher rate of return, as well as the higher risk. Assuming this restriction, it is possible to reduce the risk from 41.29% (the risk of CTT stocks) to 3.72%, through the process of diversification. Hence, assuming a maximum 10% weight of each stock, it is possible to reduce the risk.

The sensitivity of the above estimations to the exclusion of the year 2008 was also analysed, and therefore, the derivation of the EF and of the CML under an alternative first period that excludes the year 2008 was carried out.

The results are shown in Figure 6 below.

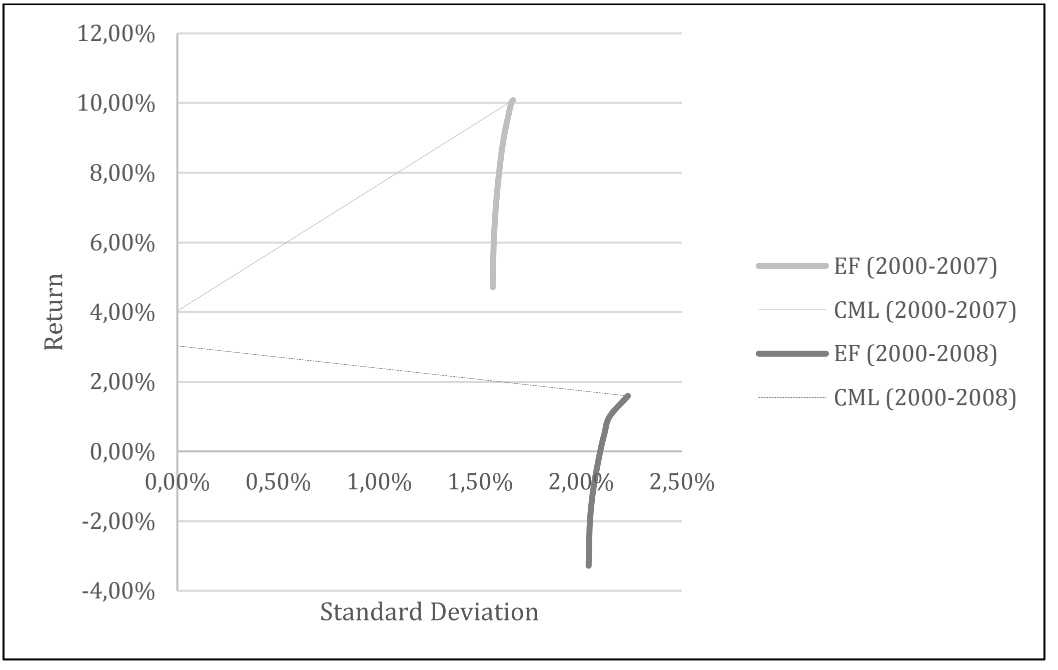

Fig. 6 - Efficient Frontier and Capital Market Lines, 2000-2007 and 2000-2008

The impact of the alternative first period is significant, as the EF moved top-left (meaning a higher expected return and a lower risk). Similarly, the estimated capital market line registers a positive slope, aligned with theory. In this context, the year of 2008 is the real booster of the bad results previously obtained in the first period. The tangent portfolio displays a return of 10.09%, and a standard deviation of 1.66%. Moreover, the GMV portfolio has a positive rate of return of 4.71%. Therefore, Figure 6 suggests that returns in 2008 drive the results for 2000-2008.

Conclusion

The concepts of the EF and the CML are relevant in portfolio construction. To build an efficient frontier and the correspondent capital market line is a natural step after having carried out a study of modern portfolio theory. However, few studies exist regarding the application of the theory to concrete capital markets, and thus this is a topic that has been neglected in the existing literature. This is the first attempt to study the Portuguese case. In addition, the study attempts to derive some conclusions regarding asset allocation decisions after the 2008 financial crisis. Indeed, it is possible to register huge differences between the two sets of the EF and the CML, according to the two periods under analysis.

During the period before 2009, the results lead to the conclusion that the best asset allocation is 100% investment in a risk-free asset with a rate of return of 3.025%. This situation is odd in the financial market framework, as the risk-free rate is above the PSI20 average rate of return of -9.04% for that same period. A further construction of the EF and the CML, considering the period between 2000 and 2007, by retrieving the year of 2008 from the data, shows that the year of 2008 is the real booster for bad results, based on data from the first period.

During the second period, very different results were obtained for the deduction of the EF. Indeed, optimal combinations are composed for at least 50% of the risky assets, yielding a rate of return higher than 5%, and a standard deviation between 2% and 3.5%. These possible asset allocations contrast with the previously recommended entire investment in the risk-free asset. A rational investor would hold a combination of risk-free assets and an optimal risky portfolio. The slope of the CML, or the SR of the market portfolio, represents an increment of 2.64% on the rate of return, considering an increment of 1% on risk. After the 2008 crisis, it seems that the stock market returned better results, showing some recovery.

This paper estimates the EF and the CML as instruments of portfolio investment decision, in the Portuguese case. However, the estimation is sensitive to the time period, as shown for the first period, when the year of 2008 is not included. This enhances the sensitivity of portfolio theory applications to the input data estimations as a guide to investors' asset allocation decisions. Further research should consider other periods, as well as the impact that different periods might have on optimal portfolios, periodically.

References

Allen, F., and Gale, D. (2000). Financial Contagion. *The Journal of Political Economy*, Vol.108, Iss.1, 1-33.

Amenc, N., Goltz, F., and Stoyanov, St. (2011). A Post-crisis Perspective on Diversification for Risk Management. An EDHEC-Risk Institute Publication, May.

Bawa, V., Elton, E.J., and Gruber, M.J. (1979). Simple Rules for Optimal Portfolio Selection in a Stable Paretian Market. *Journal of Finance*, 34, 2, 1041-1047.

Banz, R.W. (1981). The relationship between return and market value of common stocks' *Journal of Financial Economics 9*, 3-18.

BIS (2011). The impact of sovereign credit risk on bank funding conditions, CGFS Papers, No 43, Committee on the Global Financial System, July, Bank for International Settlements.

Bodie, Z., Kane, A., and Marcus, A. (2009). *Investments*, 8th Edition, Mc-Graw Hill/Irwin.

Brennan, T.J., Lo, A.W. (2011). The origin of behavior. *Quarterly Journal of Finance 1,* 55-108.

Buiter, W.H. (2003). James Tobin: An appreciation of his contribution to economics. *Economic Journal*, Royal Economic Society, vol.113 (491).

Chatelain, J. B., and Ralf, K. (2012). The failure of financial macroeconomics, and what to do about it. The Manchester School, 80(s1), 21-53.

Dowd, K (2000). Adjusting for risk: An improved Sharpe ratio. *International Review of Economics and Finance*, *9*, 209–222.

Dybvig, P.H., and Ingersoll, J.E. (1982). Mean-Variance Theory in Complete Markets. The *Journal of Business*. Vol.55, 2, 233-251.

Elton, E. J., Gruber, M. J., and Padberg, M. W. (1977). Simple Rules for Optimal Portfolio Selection: The Multi Group Case. *Journal of Financial and Quantitative Analysis*, 329-345.

Elton, E. J., Gruber, M.J. Bown, S. J., and Goetzmann, W. N. (2010). *Modern Portfolio Theory and Investment Analysis*, International Student Version, 8th Edition, John Wiley & Sons Inc., New York.

Fama, E. F. (1965). The Behavior of Stock Market Prices. *Journal of Business*, 37, January, pp. 34-105.

Fama, E. F. (1973). Risk, Return, and Equilibrium: Empirical Tests. *The Journal of Political Economy*, *81*(3), 607-636.

Fama, E. F. (1991). Efficient Capital Markets: II. *The Journal of Finance*, Vol. 46, No. 5 (Dec., 1991), 1575-1617.

Fama, E.F., and French, K.R. (1992). The Cross-section of Expected Stock Returns. *Journal of Finance*, Vol. 47, No. 2, pp. 427-465.

Fama, E. F., and French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, *33*, 3-56.

Fama, E., and French, K. R. (2004). The Capital Asset Pricing Model: Theory and Evidence. *Journal of Economic Perspectives, 18*(3), 25–46.

Feldman, D., and Reisman, H. (2001). Simple construction of the efficient frontier, Copyright © David Feldman and Haim Reisman.

Garcia, M. T. M., and Guerreiro, J. P. S. M. (2016). Internal and external determinants of banks' profitability - the Portuguese case. *Journal of Economic Studies*, Vol. 43, Issue 1, 90-107.

Girard, E., and Ferreira, F. (2005). An assets-efficient frontier guideline for investments courses, *Journal of College Teaching & Learning* 2(1), 53-65.

Jagric, T., Podobnik, B., Stasek, S., and Jagric, V. (2007). Risk-adjusted Performance of Mutual Funds: Some Tests. *South-Eastern Europe Journal of Economics*, *2*, 233-244.

Jensen, M. C. (1967). The Performance of Mutual Funds in the Period 1945-1964. *Journal of Finance*, Vol. 23, No. 2, 389-416.

Kirman, A. (2010). The economic crisis is a crisis for economic theory. CESifo Economic Studies, 56(4), 498-535.

Kolm, P. N., Tütüncü, R., and Fabozzi, F. J. (2014). 60 Years of portfolio optimization: Practical challenges and current trends. *European Journal of Operational Research*, 234 (2), 356-371.

Lai, T. L., Haipeng, X., and Chen, Z. (2009). Mean-variance portfolio optimization when means and covariances are unknown, Technical Report No. 2009-8, May 2009, Department of Statistics, STANFORD UNIVERSITY Stanford, California 94305-4065.

Levišauskait, K. (2010). Investment Analysis and Portfolio Management. *Leonardo da Vinci prog. project.*

Lintner, J. (1965). The Valuation of Risk Assets and Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics and Statistics*, Vol. 47, 13-37.

Markowitz, H. (1952). Portfolio Selection. *The Journal of Financ*e, Vol. 7, 1, 77-91.

Markowitz, H. (1959). Portfolio Selection: Efficient Diversification of Investments, *Cowles Foundation Monograph Nº16*, New York: Wiley & Sons, Inc.

Markowitz, H. (1987). Mean-Variance Analysis in Portfolio Choice and Capital Markets. New York: Basil Blackwell.

Merton, R. C. (1969). Lifetime Portfolio Selection under Uncertainty: The Continuous Time Case. *Review of Economics and Statistics*, *51*, 247-257.

Merton R.C. (1972). An Analytic Derivation of the Efficient Portfolio Frontier. *The Journal of Financial and Quantitative Analysis,* Vol. 7, No. 4 (Sep., 1972).

Michaud, R. O. (1989). The Markowitz optimization enigma: is 'optimized' optimal? *Financial Analysts Journal*, 45 (1), 31-42.

Nanto D.K. (2009). *The Global Financial Crisis: Analysis and Policy Implications*. Congressional Research Service 7-5700.

Roy, A. D. (1952). Safety First and the Holding of Assets. *Econometrica*, Vol. 20, No. 3, 431-449.

Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *Journal of Financ*e, Vol. 19, 425-442.

Sharpe, W. F. (1965). A Simplified Model for Portfolio Analysis. *Management Science*, Vol. 9.

Sharpe, W. F. (1966). Mutual Fund Performance. *Journal of Business*, 39, 119- 138.

Sharpe, W. F. (1975). The Sharpe ratio. *The Journal of Portfolio Management*.

Sharpe, W. F. (1994). The Sharpe Ratio. *The Journal of Portfolio Management*, *Fall*, 49– 58.

Tobin, J. (1958). Liquidity preference as behavior towards risk. *Review of Economic Studies*, 25, 65-86.

Treynor, J. L. (1965). How to rate Investment Fund Managers. *Harvard Business Review*, 43, No. 1, 63-75.

Treynor, J. L., and Black, F. (1973). How to Use Security Analysis to Improve Portfolio Selection. *Journal of Business*, 46, 66-86.

Appendixes

Asset	Altri	Banco BP	BCP	BES	Brisa	Cimpor	EDP	EDP Renc	GALP Ene	Jerónimc	Mota-Eng	NOS	Portucel	Portugal	REN	Semapa	Sonae SG	Sonaecor	Sonae In	Teixeira I	PSI20
Altri	0,003989																				
Banco BP	0,000264	0,001496																			
BCP	0,000530	0,000695	0,001741																		
BES	0,000405	0,000582	0,000578	0,000877																	
Brisa	0,000259	0,000374	0,000397	0,000333	0,000813																
Cimpor	0,000583	0,000395	0,000446	0,000378	0,000314	0,001408															
EDP	0,000489	0,000421	0,000615	0,000379	0,000438	0,000340	0,001349														
EDP Renc	0,000162	0,000288	0,000124	0,000130	0,000165	0,000131	0,000262	0,002770													
GALP Ene	0,000254	0,000218	0,000366	0,000256	0,000284	0,000254	0,000429	0,000431	0,003393												
Jerónimc	0,000500	0,000373	0,000529	0,000391	0,000293	0,000349	0,000648	0,000410	0,000533	0,002489											
Mota-Eng	0,000683	0,000426	0,000710	0,000450	0,000388	0,000463	0,000460	0,000080	0,000420	0,000507	0,001640										
NOS	0,000429	0,000707	0,000693	0,000554	0,000351	0,000454	0,000539	0,000222	0,000275	0,000901	0,000518	0,002699									
Portucel	0,000395	0,000431	0,000466	0,000320	0,000186	0,000247	0,000398	0,000126	0,000329	0,000480	0,000413	0,000559	0,001229								
Portugal	0,000193	0,000467	0,000556	0,000307	0,000337	0,000402	0,000529	0,000297	0,000427	0,000654	0,000367	0,000857	0,000452	0,001880							
REN	0,000238	0,000207	0,000089	0,000294	0,000168	0,000164	0,000254	0,000247	0,000464	0,000131	0,000227	0,000169	0,000245	0,000176	0,002615						
Semapa	0,000282	0,000398	0,000471	0,000280	0,000291	0,000315	0,000349	0,000131	0,000192	0,000373	0,000429	0,000394	0,000463	0,000356	0,000113	0,001274					
Sonae SG	0,000701	0,000946	0,000841	0,000751	0,000501	0,000505	0,000812	0,000145	0,000357	0,000984	0,000725	0,001313	0,000642	0,000910	0,000273	0,000520	0,002636				
Sonaecor	0,000425	0,000736	0,000914	0,000553	0,000229	0,000499	0,000509	-0,000019	0,000155	0,000573	0,000806	0,001356	0,000780	0,001218	0,000071	0,000568	0,001671	0,005495			
Sonae Inc	0,000830	0,000543	0,000604	0,000500	0,000317	0,000538	0,000543	0,000065	0,000232	0,000511	0,000689	0,000527	0,000536	0,000359	0,000164	0,000522	0,000954	0,000907	0,002194		
Teixeira I	0,000851	0,000860	0,001043	0,000712	0,000545	0,000642	0,000592	-0,000048	0,000582	0,000741	0,001009	0,000646	0,000673	0,000495	0,000267	0,000642	0,001127	0,000908	0,001025	0,002802	
PSI20	0,000404	0,000570	0,000751	0,000459	0,000410	0,000422	0,000673	0,000237	0,000443	0,000650	0,000509	0,000776	0,000438	0,000820	0,000223	0,000393	0,000923	0,000867	0,000515	0,000725	0,000687

Table A1 - Variance-Covariance Matrix (Pre 2009 period)

Asset	Altri	Banco BP	BANIF	BCP	CTT	EDP	EDP Reno	GALP Ene	Impresa	Jerónimo	Mota-Eng	NOS	Portucel	Portugal T	REN	Semapa	Sonae SG	Teixeira	PSI20
Altri	0,002252																		
Banco BPI	0,001390	0,004207																	
BANIF	0,000168	0,001096	0,033641																
BCP	0,001564	0,003165	0,000985	0,005063															
CTT	0,000119	0,000117	0,000163	0,000375	0,003279														
EDP	0,000818	0,001158	0,000042	0,000992	0,000043	0,001287													
EDP Renov	0,000843	0,000954	0,000258	0,001139	0,000010	0,000763	0,001610												
GALP Energ	0,000813	0,000907	-0,000114	0,000923	-0,000033	0,000663	0,000809	0,001872											
Impresa	0,000791	0,001085	-0,000044	0,001248	-0,000038	0,000536	0,000510	0,000443	0,003970										
Jerónimo N	0,000752	0,000719	0,000978	0,000796	-0,000071	0,000493	0,000578	0,000724	0,000279	0,001822									
Mota-Engil	0,001510	0,001916	0,000727	0,002240	0,000284	0,000983	0,000974	0,001229	0,000946	0,000855	0,003535								
NOS	0,001052	0,001318	0,000096	0,001419	0,000018	0,000878	0,000822	0,000748	0,000582	0,000456	0,001267	0,001966							
Portucel	0,000837	0,000842	0,000412	0,001067	0,000152	0,000515	0,000557	0,000604	0,000738	0,000475	0,000959	0,000605	0,001098						
Portugal Te	0,000634	0,001445	0,000612	0,001157	0,000159	0,000735	0,000627	0,000730	0,000442	0,000318	0,001410	0,000898	0,000590	0,003408					
REN	0,000456	0,000757	0,000466	0,000806	0,000067	0,000365	0,000381	0,000409	0,000363	0,000289	0,000630	0,000448	0,000386	0,000466	0,000597				
Semapa	0,000901	0,001036	0,000645	0,001032	0,000166	0,000596	0,000691	0,000675	0,000714	0,000500	0,001111	0,000723	0,000758	0,000494	0,000346	0,001346			
Sonae SGPS	0,001198	0,001481	0,000532	0,001729	-0,000025	0,000754	0,000858	0,001003	0,000815	0,000757	0,001524	0,001036	0,000775	0,000934	0,000470	0,000892	0,001845		
Teixeira Du	0,000656	0,001570	0,000845	0,001846	0,000225	0,000677	0,000559	0,000395	0,000979	0,000366	0,001263	0,000708	0,000534	0,000931	0,000460	0,000783	0,000737	0,006584	
PSI20	0,000990	0,001399	0,000495	0,001547	0,000101	0,000825	0,000838	0,000919	0,000607	0,000740	0,001308	0,000945	0,000654	0,000971	0,000440	0,000724	0,001044	0,000755	0,000988

Table A2 - Variance-Covariance Matrix (Post 2009 period)

Asset	Altri	Banco BPI	BCP	BES	Brisa	Cimpor	EDP	GALP Ener	Jerónimo	Mota-Engi	NOS	Portucel	Portugal T	REN	Semapa	Sonae SGF	Sonaecom	Sonae Ind	Teixeira D	PSI20
Altri	0,003465																			
Banco BPI	3,67E-05	0,000929																		
BCP	0,000243	0,000295	0,001167																	
BES	8,83E-05	0,000201	0,00023	0,000416																
Brisa	3,74E-05	0,000181	0,000171	0,000105	0,000534															
Cimpor	0,000212	0,000151	0,000176	0,00015	0,000168	0,001021														
EDP	0,000156	0,00022	0,000442	0,000159	0,000266	0,000122	0,000934													
GALP Ener	3,64E-06	-3E-08	3,84E-05	5,19E-05	5,14E-05	7,24E-05	0,000102	0,002863												
Jerónimo	0,000251	0,000321	0,000371	0,000259	0,000198	0,000205	0,000269	9,62E-05	0,00188											
Mota-Engi	0,000292	0,000161	0,000368	0,000159	0,000179	0,000159	0,000241	4,77E-05	0,000288	0,001282										
NOS	8,34E-05	0,000499	0,00049	0,000331	0,000161	0,000166	0,000363	4,91E-05	0,000751	0,000274	0,002448									
Portucel	0,000198	0,000259	0,000299	0,000217	0,000135	0,000197	0,000277	3,15E-05	0,00031	0,000217	0,000494	0,001011								
Portugal T	1,92E-05	0,000322	0,000375	0,000126	0,000234	0,000167	0,000369	1,7E-05	0,000421	0,000122	0,000687	0,000367	0,001583							
REN	1,64E-05	2,14E-05	-6,8E-05	0,000144	5,97E-05	3,91E-05	6,99E-05	0,000252	-8,7E-05	5,16E-05	2,34E-05	0,000133	4,56E-05	0,002734						
Semapa	9,81E-05	0,000264	0,000347	0,000189	0,000224	0,00025	0,00026	5,69E-06	0,000309	0,000284	0,000319	0,00034	0,000276	2,14E-05	0,001234					
Sonae SGF	0,000242	0,000524	0,000588	0,000378	0,000295	0,000293	0,000534	9,04E-05	0,000822	0,000457	0,001123	0,000441	0,000874	3,57E-05	0,000405	0,002161				
Sonaecom	0,000239	0,000508	0,000671	0,000368	0,000219	0,000291	0,000449	-5,3E-05	0,000588	0,000515	0,001334	0,000644	0,001218	-6,2E-06	0,000504	0,001604	0,005516			
Sonae Ind	0,000282	0,000165	0,000273	0,000142	0,000153	0,000197	0,000276	-3,4E-05	0,000354	0,000311	0,00024	0,000308	0,000227	-4,8E-05	0,000331	0,000487	0,000613	0,00169		
Teixeira D	0,000308	0,00035	0,000463	0,000207	0,000222	0,000329	0,000282	5,92E-05	0,000459	0,000516	0,000245	0,000377	0,000253	-2,3E-05	0,000441	0,000563	0,000515	0,000452	0,001808	
PSI20	0,00013	0,000332	0,000488	0,000211	0,00023	0,000204	0,000425	7,83E-05	0,000402	0,000242	0,000591	0,000302	0,000624	5,6E-05	0,000294	0,000702	0,000784	0,000256	0,000334	0,000456

Table A3 - Variance-Covariance Matrix (Pre 2008 period)

Portfolio Statistics	GMV						Market
Rp	-3,28%	-2,00%	-1,00%	0,00%	0,50%	1,00%	1,59%
σp	2,04%	2,04%	2,06%	2,10%	2,12%	2,14%	2,23%
Sharpe Ratio	-1,610334	-0,9781946	-0,484616	2,0027E-07	0,23628742	0,46686387	0,71337733
Weights							
Altri	2,35%	4,47%	6,62%	8,76%	9,83%	10,00%	10,00%
Banco BPI	5,00%	4,04%	2,65%	1,27%	0,57%	0,00%	0,00%
BCP	0,41%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
BES	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	0,00%
Brisa	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
Cimpor	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
EDP	6,15%	5,27%	4,05%	2,82%	2,21%	1,00%	0,00%
EDP Renovaveis	9,81%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
GALP Energia	3,64%	4,82%	6,19%	7,55%	8,24%	10,00%	10,00%
Jerónimo Martins	1,78%	1,83%	1,79%	1,76%	1,75%	2,09%	10,00%
Mota-Engil	4,84%	4,98%	4,88%	4,77%	4,71%	5,68%	10,00%
NOS	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Portucel	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
Portugal Telecom	5,26%	4,59%	3,83%	3,07%	2,69%	1,23%	0,00%
REN	9,54%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
Semapa	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
Sonae SGPS	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Sonaecom	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Sonae Industria	1,22%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Teixeira Duarte	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

Table A4 - Portfolio performance (Pre 2009 period)

Portfolio Statistics					GMV						Market	
Rp	1,00%	2,00%	3,00%	4,00%	4,24%	5,00%	6,00%	7,00%	8,00%	9,00%	9,80%	9,86%
σp	3,55%	3,54%	3,53%	3,53%	3,53%	3,53%	3,54%	3,55%	3,57%	3,61%	3,72%	3,83%
Sharpe Ratio	0,2815184	0,5647803	0,8488701	1,1328473	1,2009835	1,4155735	1,6958462	1,9709978	2,23794108	2,49363905	2,63534901	2,57545772
Weights												
Altri	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,27%	2,83%	5,77%	9,12%	10,00%	10,00%
Banco BPI	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
BANIF	1,40%	1,21%	1,02%	0,84%	0,79%	0,66%	0,43%	0,00%	0,00%	0,00%	0,00%	0,00%
BCP	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
CTT	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
EDP	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
EDP Renováveis	10,00%	9,73%	9,45%	9,25%	9,30%	9,46%	9,53%	7,86%	5,44%	2,18%	0,00%	10,00%
GALP Energia	6,20%	6,55%	6,97%	7,47%	7,70%	8,42%	9,31%	8,61%	7,56%	6,19%	10,00%	10,00%
Impresa	6,22%	5,77%	5,31%	4,88%	4,82%	4,61%	4,21%	2,80%	0,96%	0,00%	0,00%	0,00%
Jerónimo Martins	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
Mota-Engil	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
NOS	0,96%	1,81%	2,62%	3,52%	3,86%	4,92%	6,25%	5,60%	4,61%	3,22%	10,00%	10,00%
Portucel	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
Portugal Telecom	9,59%	7,75%	5,92%	4,04%	3,53%	1,92%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
REN	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	0,00%	0,00%
Semapa	5,64%	7,17%	8,71%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
Sonae SGPS	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	2,31%	5,65%	9,30%	10,00%	10,00%
Teixeira Duarte	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	0,00%

Table A5 - Portfolio performance (Post 2009 period)

Portfolio Statistics					GMV							Market
Rp	1,00%	2,00%	3,00%	4,00%	4,71%	5,00%	6,00%	7,00%	8,00%	9,00%	10,00%	10,09%
σp	1,60%	1,58%	1,57%	1,56%	1,56%	1,56%	1,57%	1,58%	1,59%	1,62%	1,65%	1,66%
Sharpe Ratio	0,624995	1,2631784	1,9091991	2,5565465	3,0125104	3,1983177	3,8269375	4,4360311	5,01785624	5,56387696	6,04439524	6,06046195
Weights												
Altri	0,55%	1,47%	2,42%	3,54%	4,28%	4,60%	5,70%	6,81%	8,08%	9,61%	10,00%	10,00%
Banco BPI	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
BCP	7,48%	6,40%	5,35%	3,99%	3,07%	2,68%	1,33%	0,00%	0,00%	0,00%	0,00%	0,00%
BES	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
Brisa	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
Cimpor	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%	10,00%
EDP	8,64%	8,43%	8,25%	7,95%	7,77%	7,68%	7,40%	7,09%	6,15%	5,30%	2,71%	0,00%
GALP Energia	5,13%	5,73%	6,32%	6,99%	7,46%	7,66%	8,34%	9,03%	9,82%	10,00%	10,00%	10,00%
Jerónimo Mar	1,59%	1,75%	1,73%	1,69%	1,60%	1,57%	1,46%	1,38%	1,25%	0,93%	1,06%	1,13%
Mota-Engil	5,91%	6,33%	6,76%	7,20%	7,51%	7,63%	8,07%	8,50%	8,67%	9,11%	10,00%	10,00%
NOS	1,60%	0,86%	0,12%	0,12%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Portucel	6,42%	6,40%	6,48%	6,32%	6,24%	6,19%	6,03%	5,86%	5,41%	5,15%	6,22%	8,87%
Portugal Telec	5,31%	5,20%	5,09%	4,75%	4,49%	4,39%	4,02%	3,63%	2,95%	2,29%	0,00%	0,00%
REN	6,93%	7,07%	7,22%	7,39%	7,51%	7,56%	7,73%	7,90%	8,16%	8,57%	10,00%	10,00%
Semapa	2,69%	3,25%	3,74%	4,29%	4,73%	4,91%	5,50%	6,08%	6,64%	7,22%	10,00%	10,00%
Sonae SGPS	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Sonaecom	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
Sonae Industr	7,77%	7,12%	6,51%	5,78%	5,33%	5,12%	4,43%	3,72%	2,86%	1,81%	0,00%	0,00%
Teixeira Duart	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%

Table A6 - Portfolio performance (Pre 2008 period)